Flame Instabilities in Type Ia Supernovae

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in collaboration with

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Type Ia Supernovae Observations

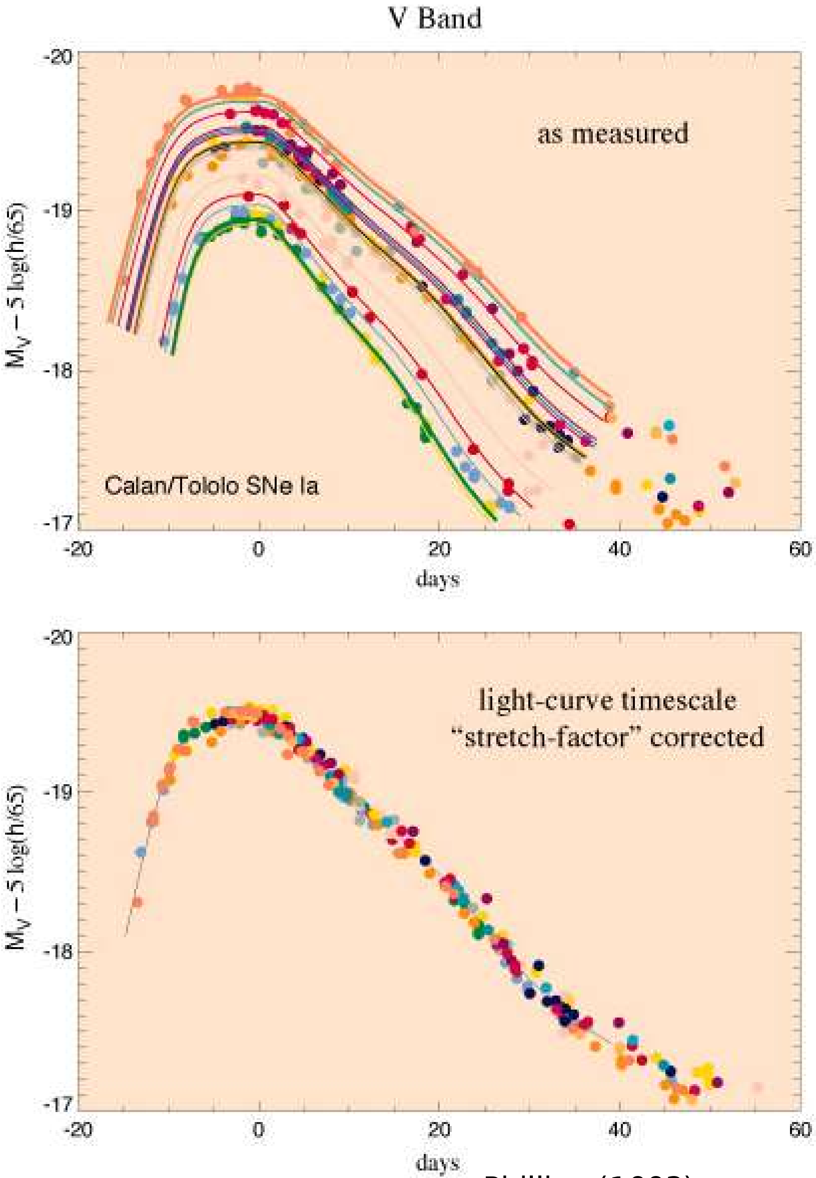
- Brightness rivals that of host galaxy, L ~ 10^{43} erg s^{-1}
- No H seen in spectra, but strong Si, Ca, and Fe lines
- Occur in old stellar populations
- Less frequent than SNe II
- Large amounts of ^{56}Ni produced
  - Radioactivity powers the lightcurve
- No compact remnant

SN 1994D (High-Z SN Search team)

SN 1998dh

Type Ia Supernovae Observations

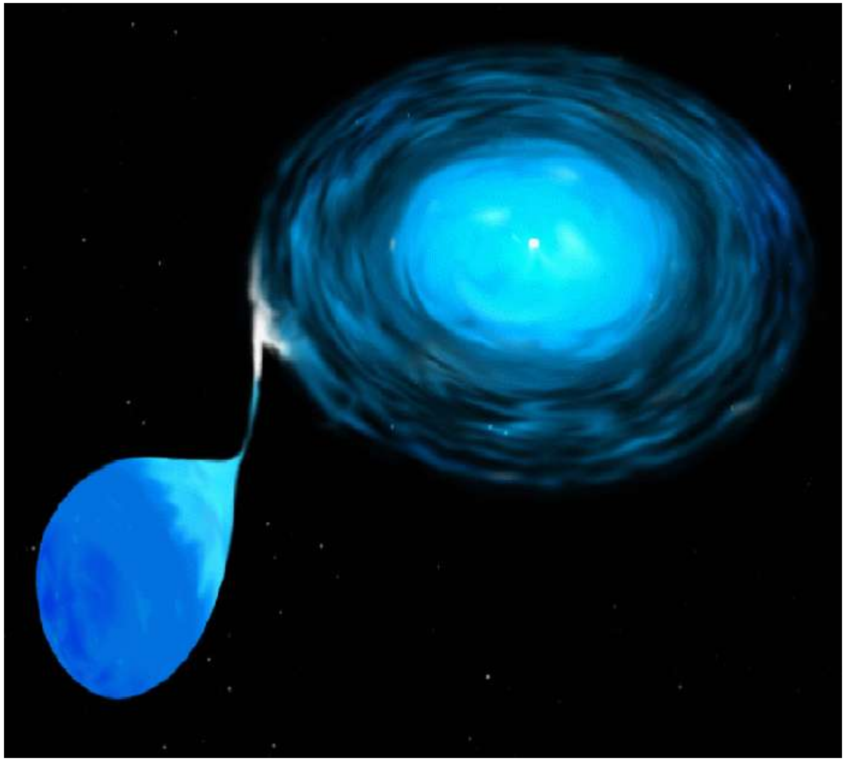
- Variation in SNe Ia lightcurves can be corrected for
  - “normalizable” standard candle
  - Broader = Brighter
  - Single parameter function
- What makes them such robust explosions?



Phillips (1993), Perlmutter et al. (1997)

Type Ia Supernovae Theory

- Thermonuclear explosion of M_{Ch} white dwarf
  - Accretes from companion at high rate
  - As M_{wd} nears M_{Ch}, convection occurs throughout interior
- Ignition near center
  - Degeneracy decouples *P* from *T*, allowing for explosive runaway
  - C+C reaction rate is very temperature sensitive.
- Burning can proceed as deflagration or detonation.



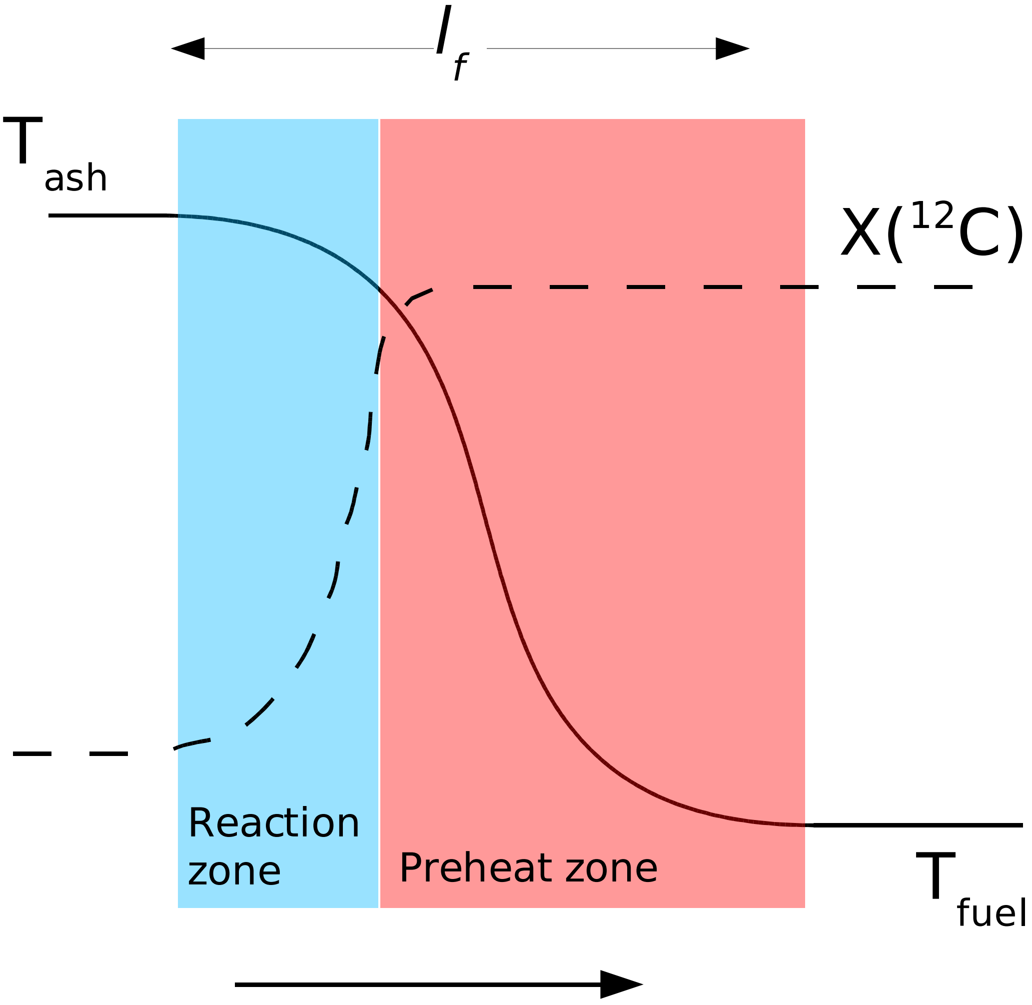
Type Ia Supernovae Theory

- Ra ~ 10^{25} (buoyancy to diffusion forces)
  - Nature of convection is not well known in this regime.
- Re ~ 10^{14} (inertial to viscous forces)
- Pr ~ 10^{-4} (momentum transport to heat conduction)
  - Viscosity effects are unimportant.
- Le ~ 10^{7} (energy transport to mass transport)
  - Mass diffusion can be neglected.
  - Large departure from typical Le ~ 1 terristrial flames.

Wunsch, Woosley, Kuhlen (2003)

Flames

- Begins as a deflagration
  - Subsonic burning front
    - Pressure is continuous across the front
    - Density drops in the ash region.
  - Thermal diffusion transports the heat
- Laminar speed too slow
  - Must accelerate considerably at low densities.
  - May transition to detonation



Explosion Requirements

- Flame must accelerate to ~ 1/3 c_s.
- Must produce intermediate mass elements (Si, S, Ar, Ca).
- Produces ~ 0.6 $M_\odot$ ^{56}Ni.
- How does the flame accelerate?
  - Flame instabilities (Landau-Darrieus, Rayleigh-Taylor)
  - Interaction with turbulence.

Increase surface area ⇒ increase flame speed.

SNe Ia Unstable Flames

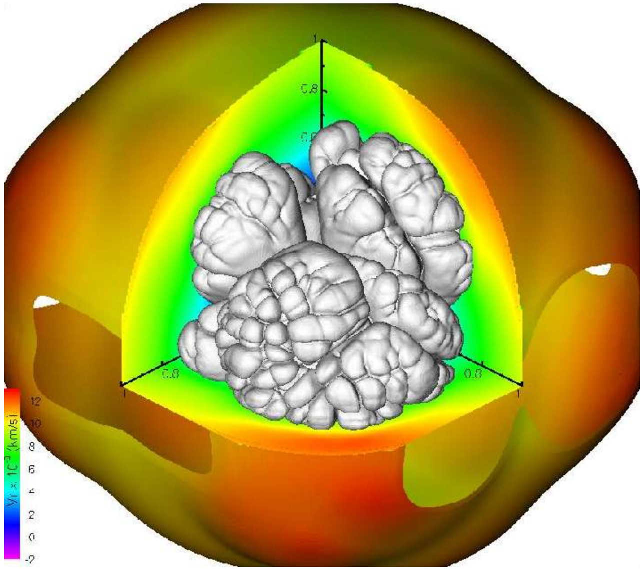
- Explosion begins as a flame in the interior of the white dwarf.
  - ~ 100 years of convection preceed ignition
  - subsonic propagation allows the star to expand.
- Hot ash is less dense than the cool fuel.
- Subjected to numerous instabilities.

Increase surface area ⇒ increase flame speed.

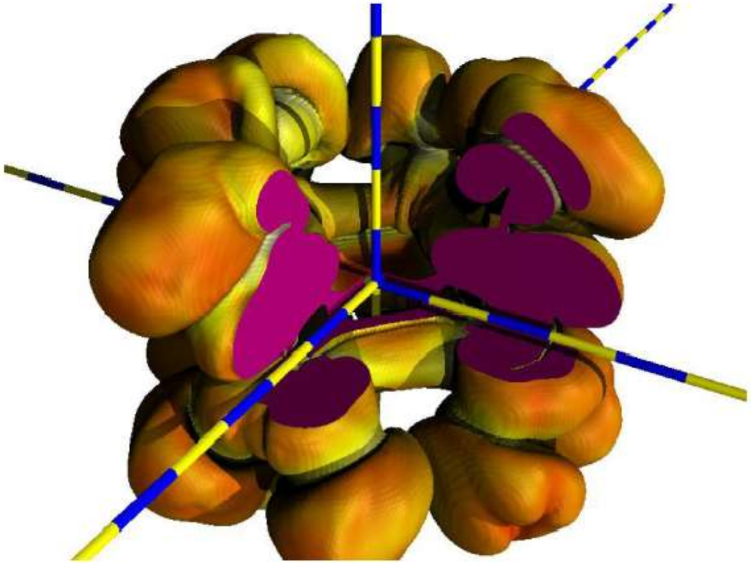
We want to understand the physics of the flame and how the combustion process changes as the explosion evolves.

Large Scale Simulations

- Instabilities are the dominant acceleration mechanism.
- Pure deflagrations can unbind the star.



Gamezo et al. (2003)

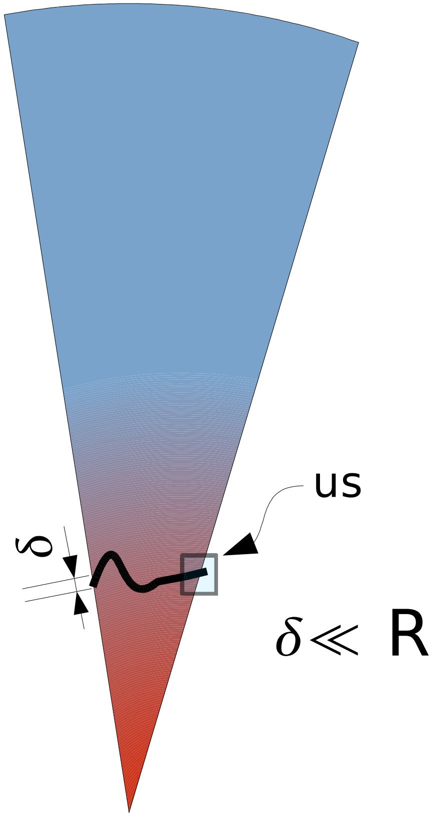


Reinecke et al. (2003)

- Some flame model is required.
  - Stellar scale ~ 10^8 cm
  - Flame width ~ 10^{-5} – 10 cm

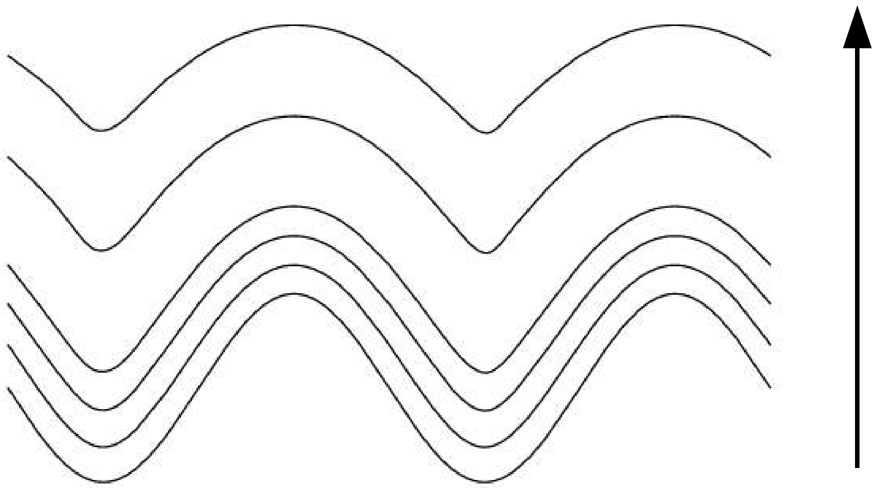
Bottom-Up Approach

- Simulations cannot resolve the star and the flame.
- We resolve the thermal structure of the flame and work up to large scales
  - Parameter free.
  - Resolved calculations can be used to validate flame models.
- Look for scaling relations that will act as subgrid models.



Landau-Darrieus Instability

- Landau-Darrieus
  - Planar flame is unstable to wrinkling/cusping due to expansion across the flame.
- Growth rate can be computed in the linear regime:



Flame cusping (Dursi et al. 2003)

$$\omega = kU_l \frac{\alpha}{\alpha+1}\left[\sqrt{\alpha - \frac{1}{\alpha} + 1 + k\, l_f \mathrm{Ma}(k\, l_f \mathrm{Ma} + 2\alpha)} - 1 + k\, l_f \mathrm{Ma}\right]$$

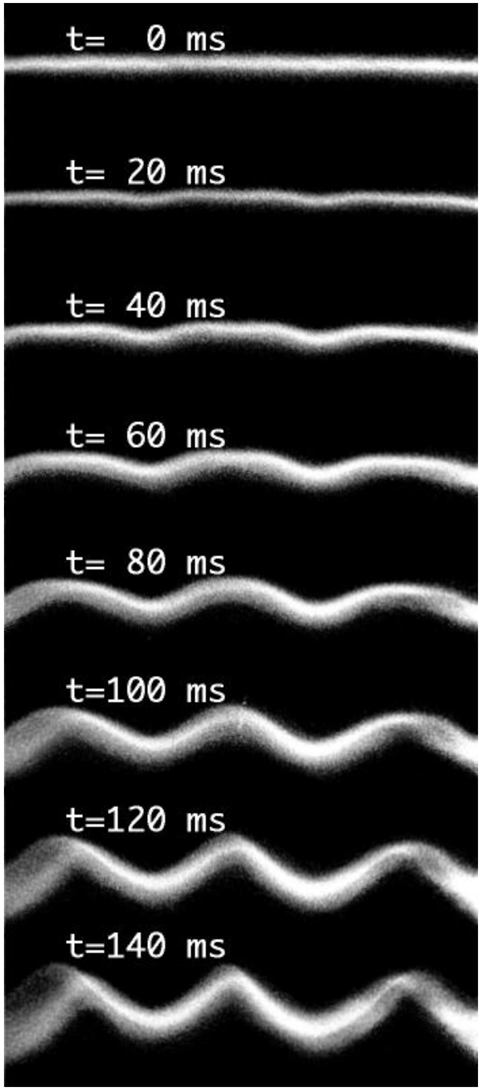
$$U = U_l\left(1 + l_f \mathrm{Ma}\frac{\partial^2 y_f}{\partial x^2}\right)$$

Zeldovich et al. (1985)

- Finite flame thickness sets a small scale cutoff.

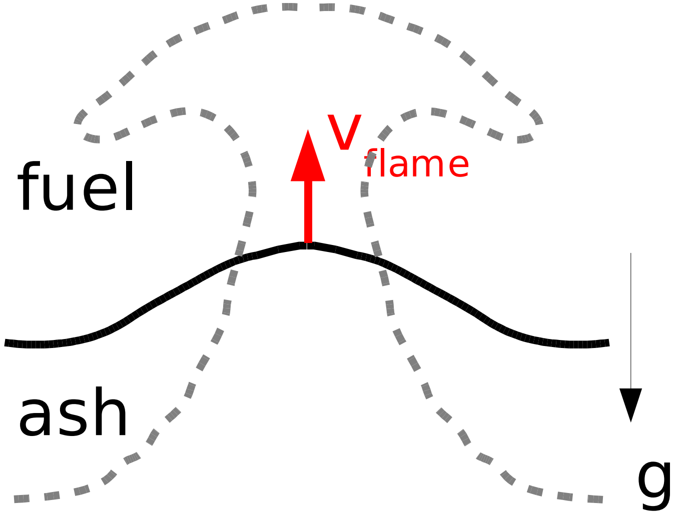
Landau-Darrieus Instability

- Well studied for terrestrial flames.
  - Growth rate confirmed experimentally
- Does non-linear cusp formation break down?
  - Active turbulent combustion?
- Provides a means to quantify curvature effects.
- Useful for code validation.



Clanet & Searby 1998

Rayleigh-Taylor Instability



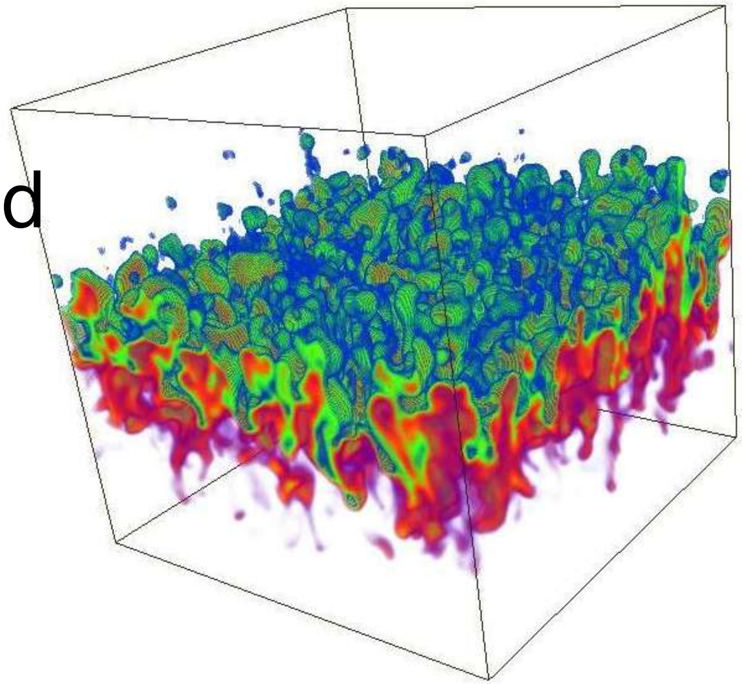
$\rho_{ash} < \rho_{fuel}$

- Rayleigh-Taylor
  - Buoyancy driven instability in presence of gravitation field.
  - Large amounts of surface area are generated.

- Well studied experimentally and numerically
  - Bubble merger model (Sharp-Wheeler) predicts growth of mixed region:

$$h = \alpha A g t^2$$

  - Measured α values range from 0.03 - 0.08



Calder et al. (2002)

Rayleigh-Taylor Instability

- Reactions set a small scale cutoff to the growth of the instability
  - Equate the growth rate of the RT instability to the timescale for a laminar flame to burn across that region

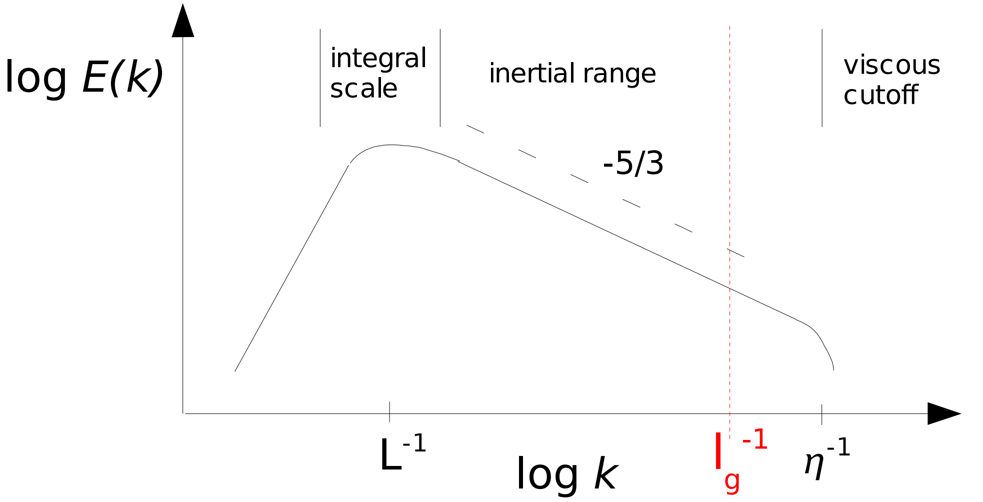
$$\omega^2 = gkA$$

$$\lambda_{\mathrm{fp}} = 4\pi \frac{v_{\mathrm{laminar}}^2}{g_{\mathrm{eff}}}$$

  - Wavelengths smaller than this will burn away.
  - At low densities, RT will dominate

Turbulence

- Cascade of kinetic energy over a range of length scales
  - Integral scale, L, where the bulk of the kinetic energy exists
  - Kolmogorov scale, η, where inertial and viscous effects balance
  - Gibson scale, l_g, where eddy can turn over before burning away.
- Size of l_g in comparison to flame width will determine the flame regime.



adapted from Peters (2000)

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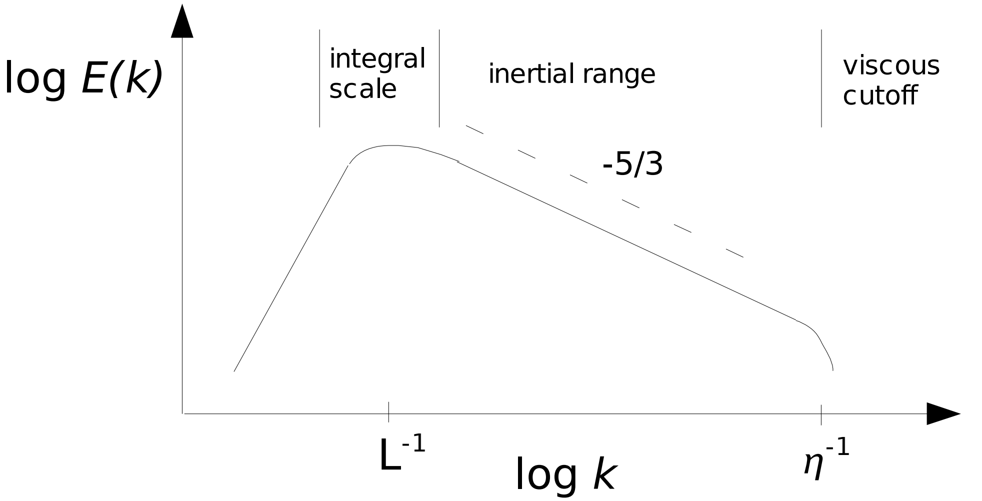
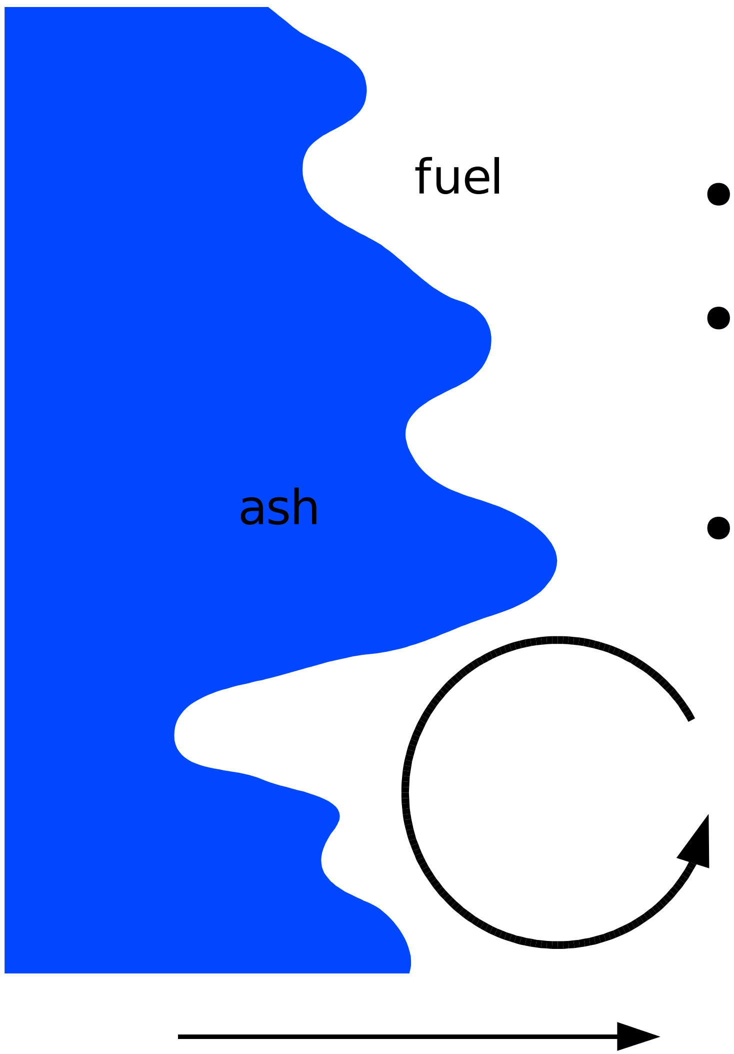


adapted from Peters (2000)

- Large Reynolds number: $\mathrm{Re} = u'L/\nu$
- Kolmogorov: kinetic energy flux is constant

$$\epsilon = u'(l)^3/l \quad \rightarrow \quad u'(l) = u'(l/L)^{1/3} \quad \rightarrow \quad L/\eta = \mathrm{Re}^{3/4}$$

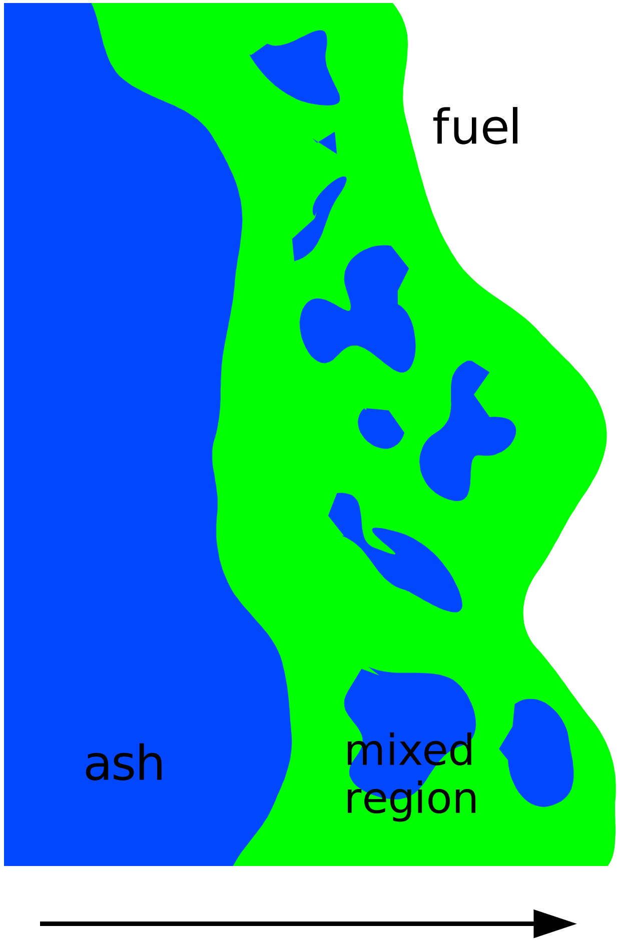
Flamelet Regime



- Flame is thinner than all turbulent scales
- Flame is a continuous surface
- Laminar propagation normal to the surface
- Turbulence serves solely to wrinkle the flame, increasing the area

Only fuel and ash exist, with a sharp interface between.

Distributed Burning Regime



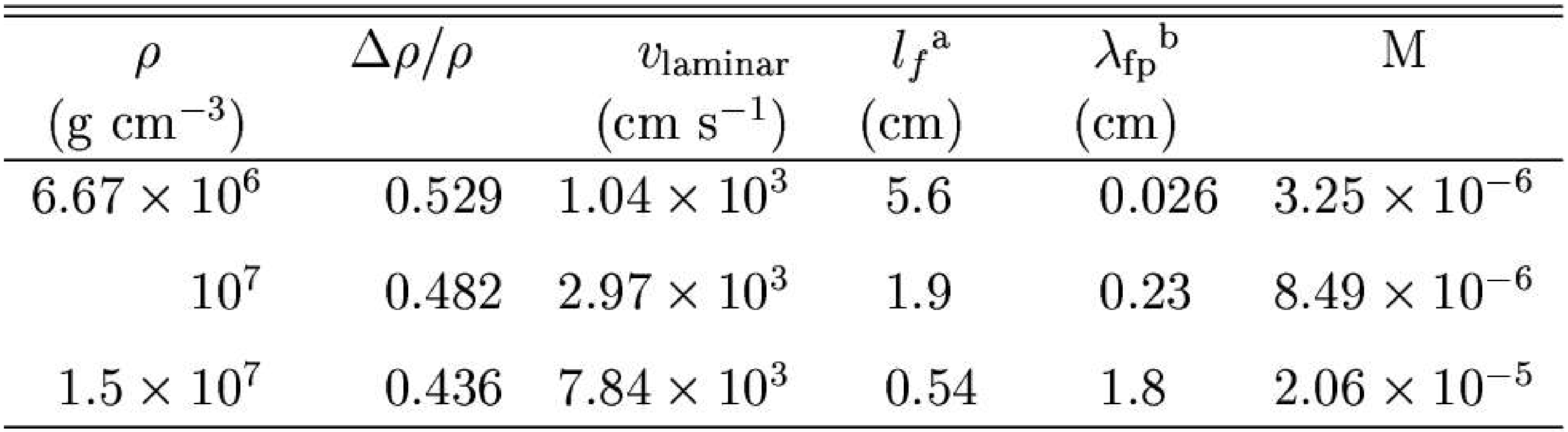
- Turbulence disrupts the flame
  - Gibson scale is thinner than the flame
- Mixed region of fuel + ash develops
- May be possible to quench the flame

Laminar flame properties suggest transition to the distributed regime at 10^7 g cm^{-3}

Niemeyer & Woosley (1997)
Niemeyer & Kerstein (1997)

This is something we can confirm

Low Density Flame Properties



ρ (g cm^{-3})	$\Delta\rho/\rho$	v_{laminar} (cm s^{-1})	l_f [a] (cm)	λ_{fp} [b] (cm)	M
6.67×10^6	0.529	1.04×10^3	5.6	0.026	3.25×10^{-6}
10^7	0.482	2.97×10^3	1.9	0.23	8.49×10^{-6}
1.5×10^7	0.436	7.84×10^3	0.54	1.8	2.06×10^{-5}

- Laminar flame speeds are very slow, $\mathrm{M} \ll 1$
- Expansion ~ 2x behind the flame.
- Densities around 10^7 g cm^{-3} pass through the region where

$$\lambda_{\mathrm{fp}} = l_f$$

Low Mach Number Hydrodynamics

- Laminar flames are very subsonic ($M \sim 10^{-5}$–10^{-6}).
- Compressible hydro is too expensive.
  - Timestep is limited by sound crossing across zone.
  - Many timesteps ⇒ large accumulation of error.
- Fuel and ash states are nearly incompressible.
  - Expansion across the flame links the two states.
  - Hydrodynamic method can exploit this to more efficiently evolve the flow.

Low Mach Number Hydrodynamics

- Low Mach number formulation projects out the compressible components.
  - Pressure decomposed into thermodynamic and dynamic components.

$$p(x,t) = p_0(t) + Mp_1(t) + M^2\pi(x,t)$$

  - Elliptic constraint provided by thermodynamics.

$$0 \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial \rho}\frac{D\rho}{Dt} + \frac{\partial p}{\partial T}\frac{DT}{Dt} + \sum_k \frac{\partial p}{\partial X_k}\frac{DX_k}{Dt}$$

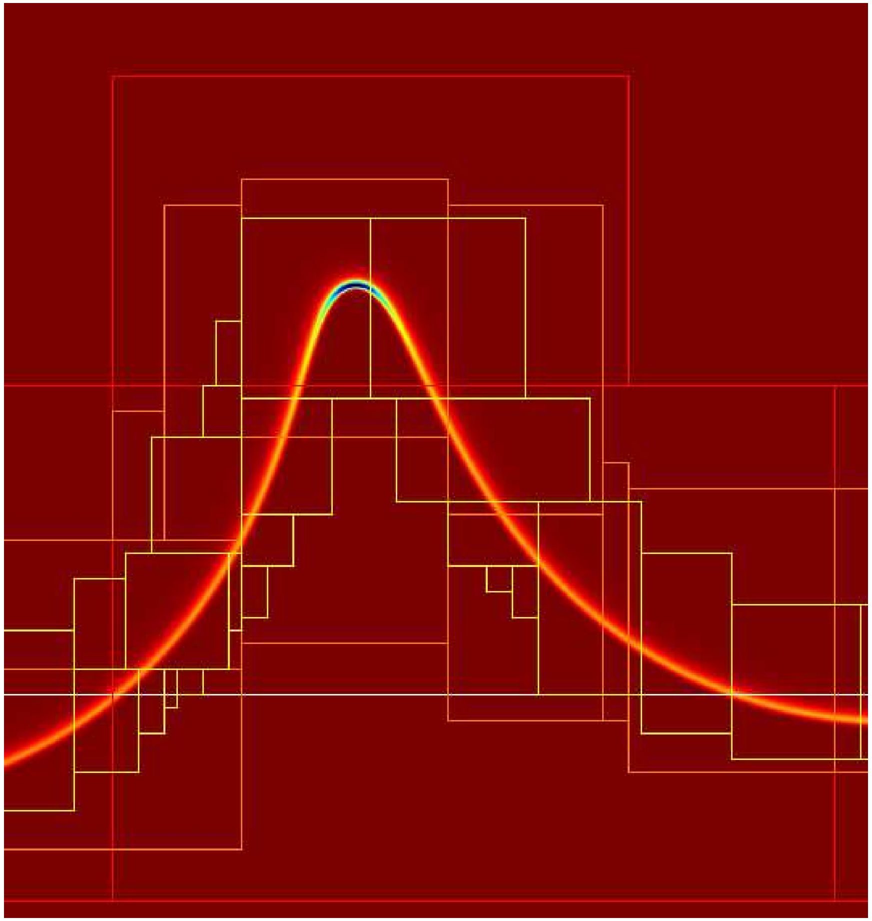
$$\nabla \cdot U = \frac{1}{\rho \frac{\partial p}{\partial \rho}}\left(\frac{\partial p}{\partial T}\frac{DT}{Dt} + \sum_k \frac{\partial p}{\partial X_k}\frac{DX_k}{Dt}\right)$$

  - Advection/Projection/Reaction formulation solves system.
  - Timestep limited by |v| and not |v| + c.

Bell et al. 2003, JCP

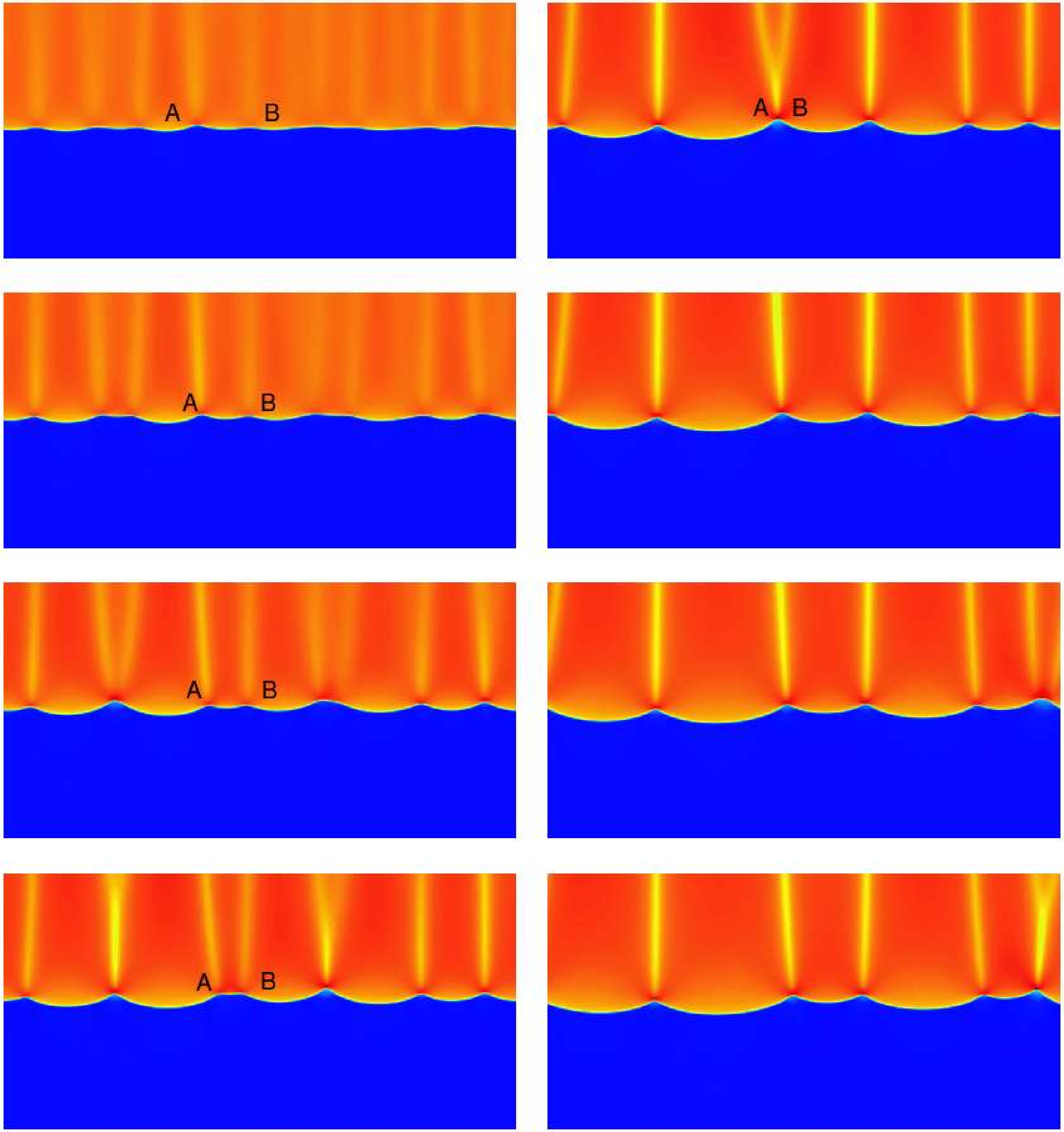
Simulation Method

- Degenerate/Relativistic EOS used.
- Single step $^{12}C+^{12}C$ rate
- Initialized by mapping 1-d steady-state laminar flame onto grid.
  - Comoving frame
- Resolution chosen to put 5-10 zones inside thermal width.
- Block-structured adaptive mesh refinement is used.



Landau-Darrieus Results

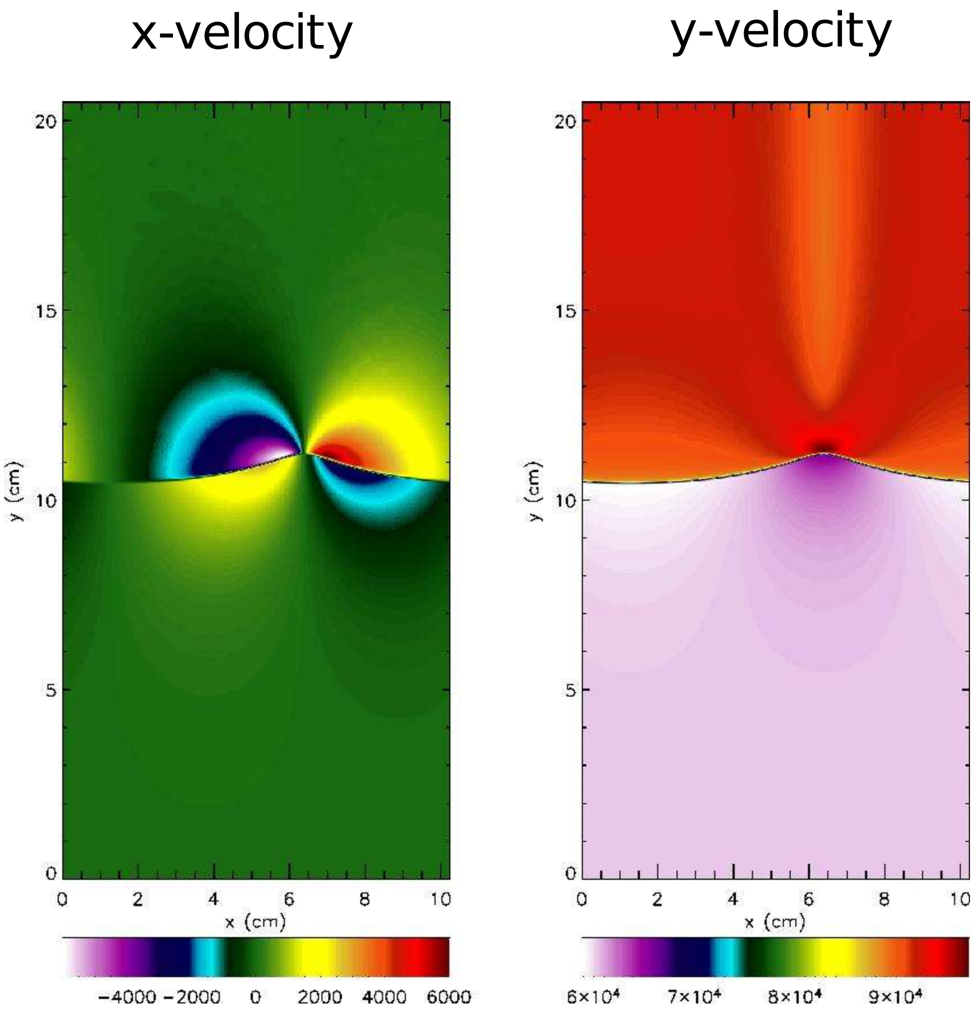
- Multimode perturbations merge into a single cusp
  - Single mode can be used to understand the physics.



Bell et al. 2003

Landau-Darrieus Results

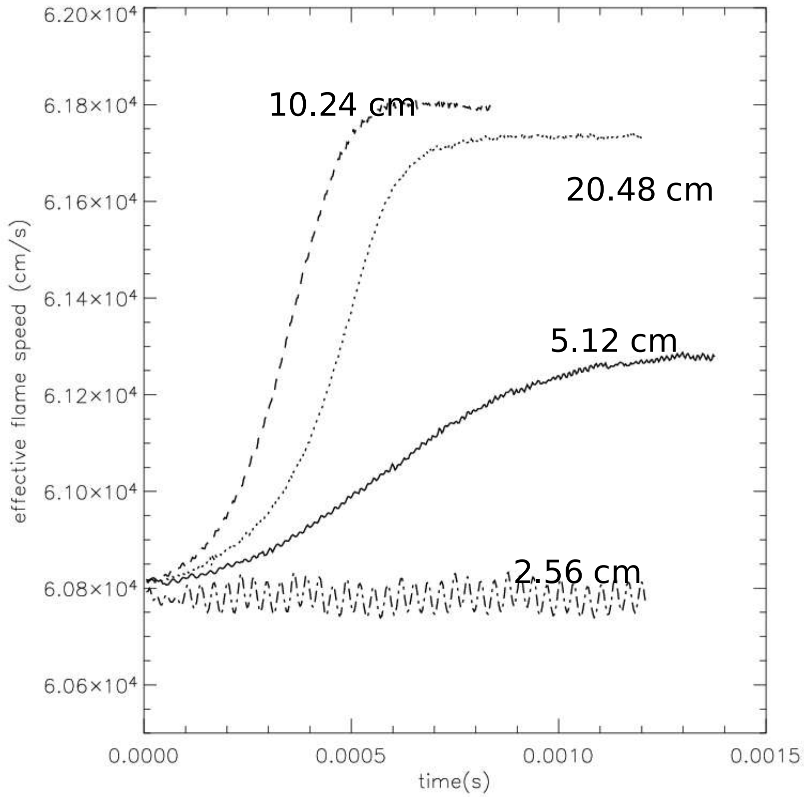
- Single mode study
  - Range of densities (2x10^{7} to 8x10^{7} g cm^{-3})
  - Varying box width
- Well defined cusps form and persist
  - No breakdown in the non-linear regime observed.
- Accelerations of a few % observed.



Bell et al. 2003, astro-ph/0311543

Landau-Darrieus Results

- Confirmed small scale cutoff to growth of LD.
    - Growth rates for different mode perturbations match theoretical prediction.



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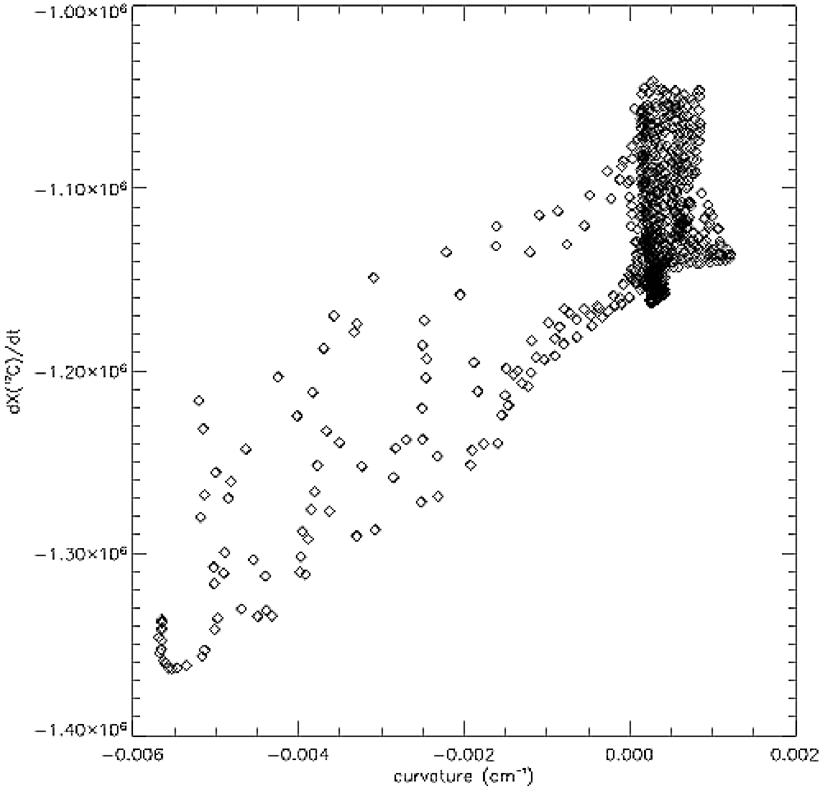
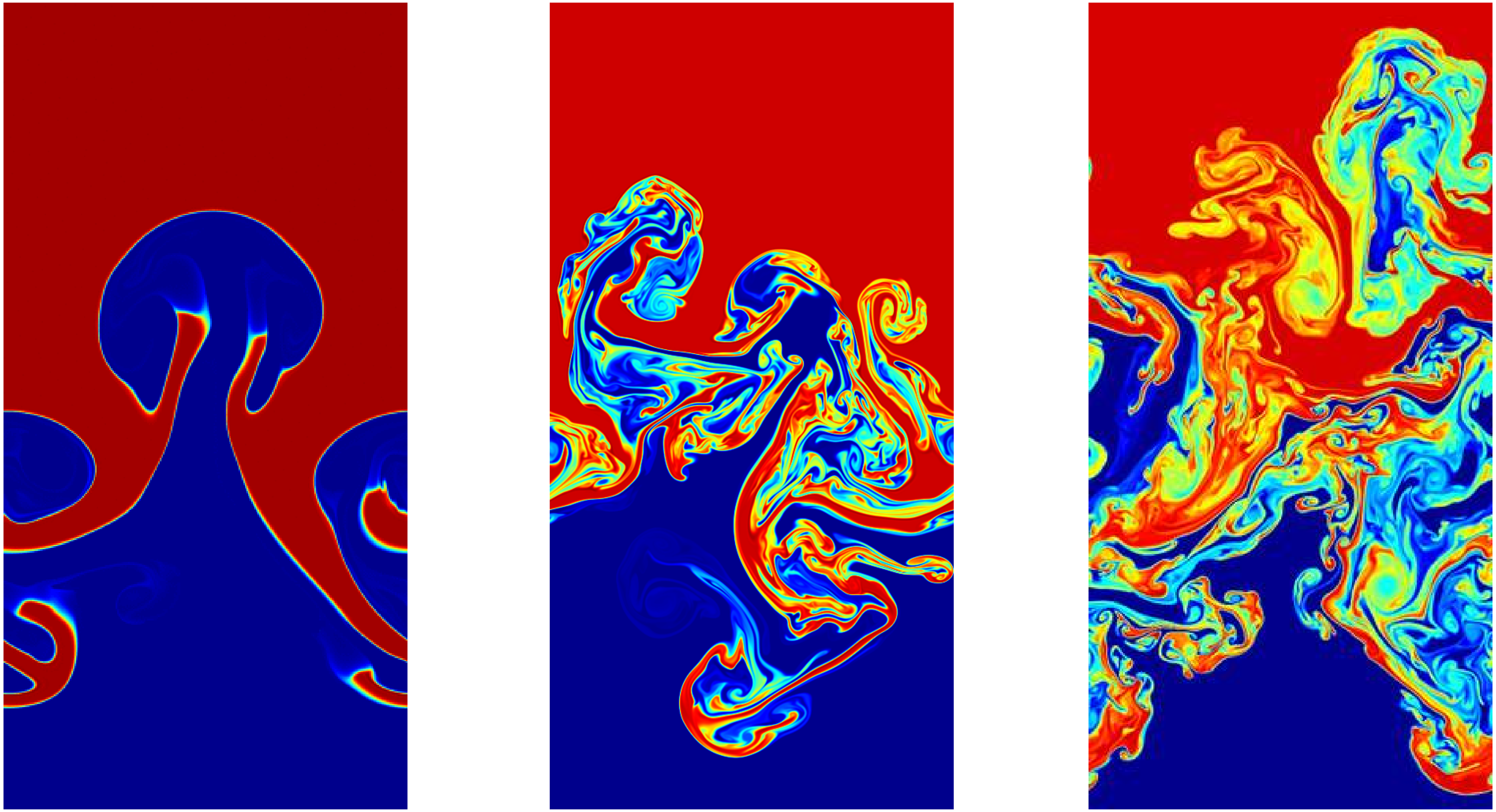


- Curvature effects quantified
    - |Ma| ~ 2
    - Agreement with compressible calculations Dursi et al. (2003)

Transition to Distributed Burning

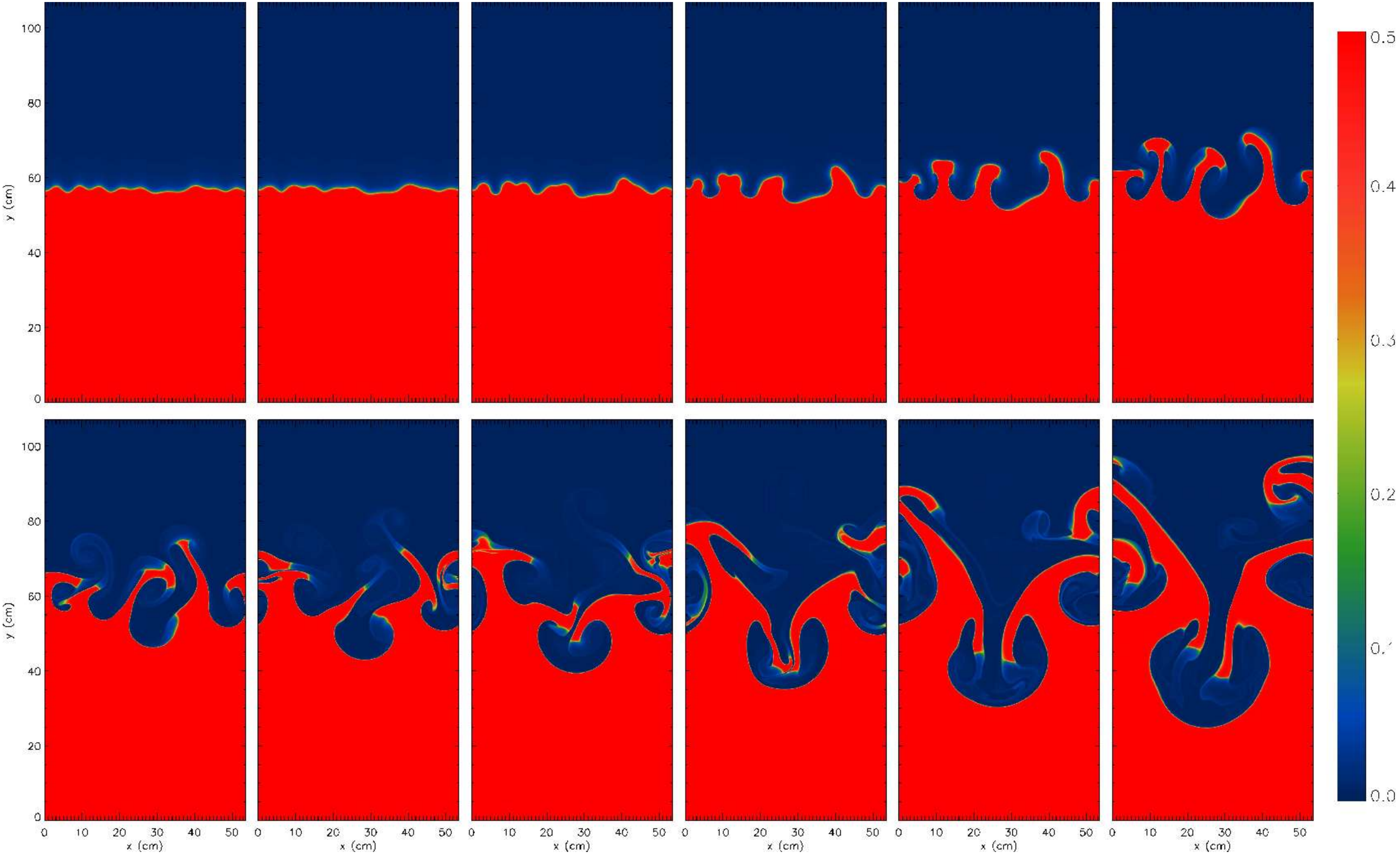


Bell et al. 2003, astro-ph/0401247

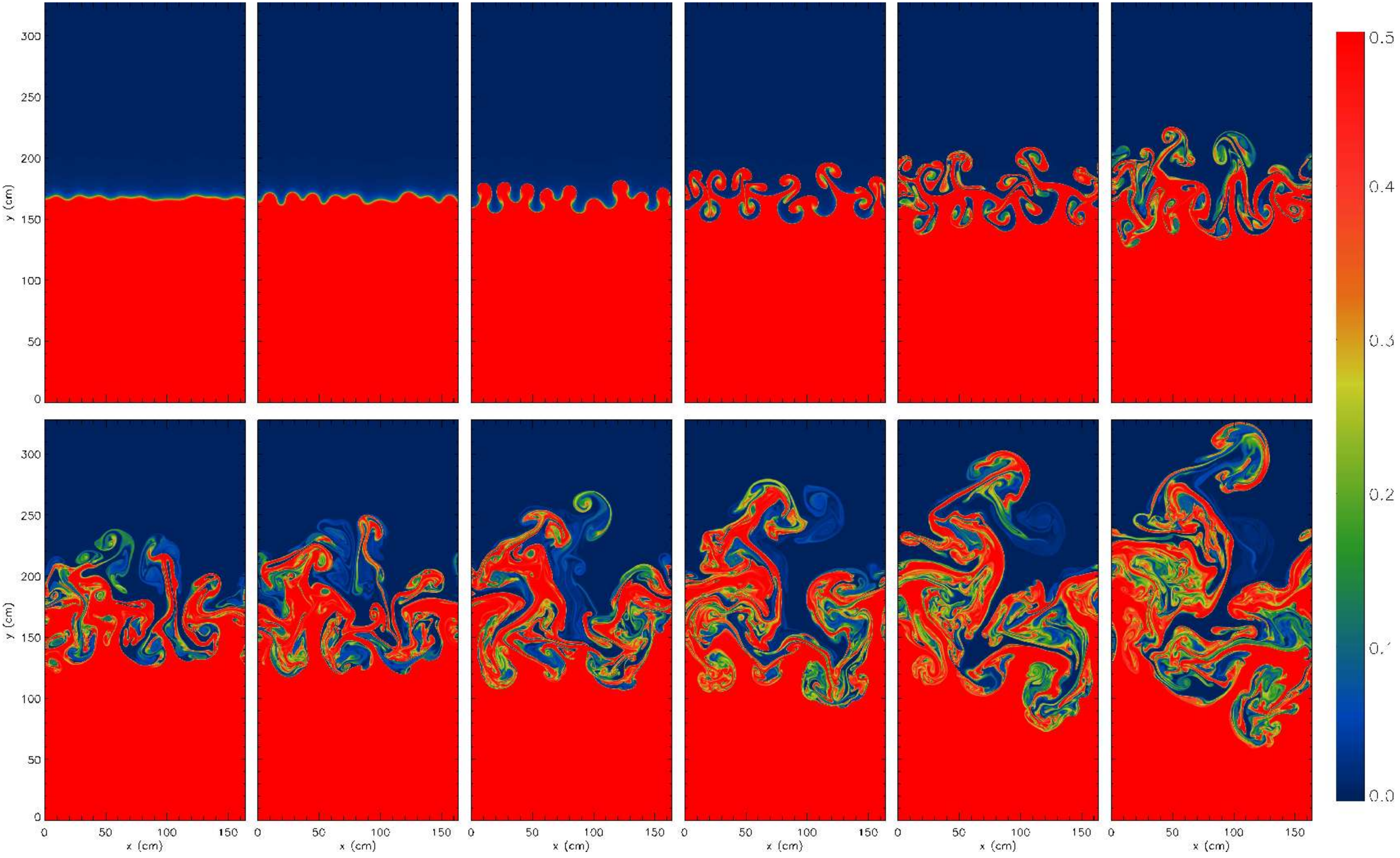
$\longleftarrow \rho$

- As ρ decreases, RT dominates over burning.
- At low ρ, flame width is set by mixing scale.

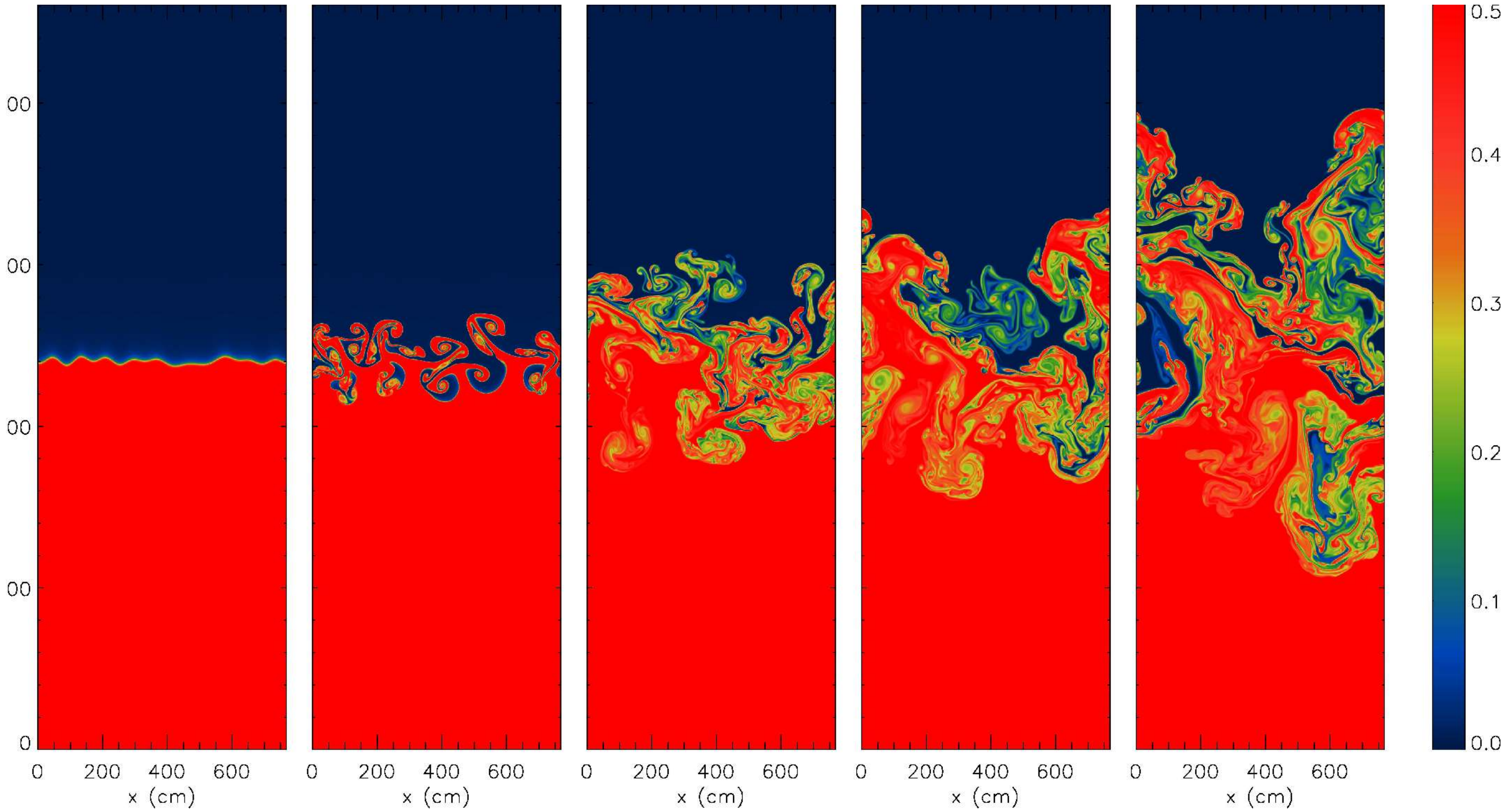
$1.5\text{x}10^7$ g cm^{-3}



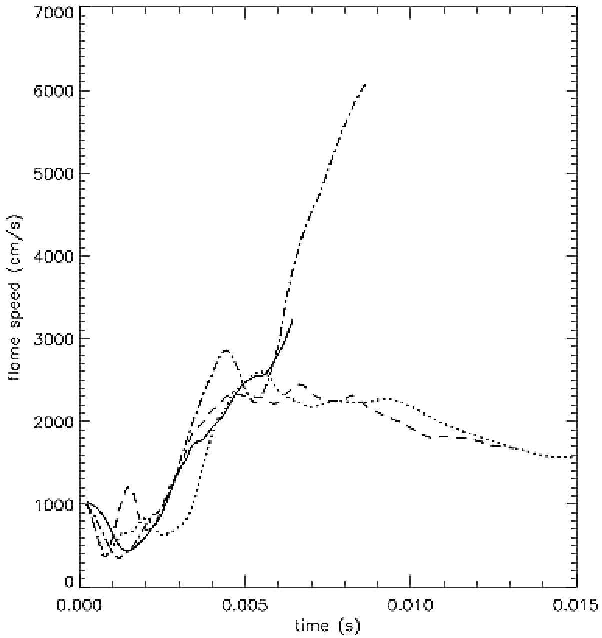
10^7 g cm^{-3}

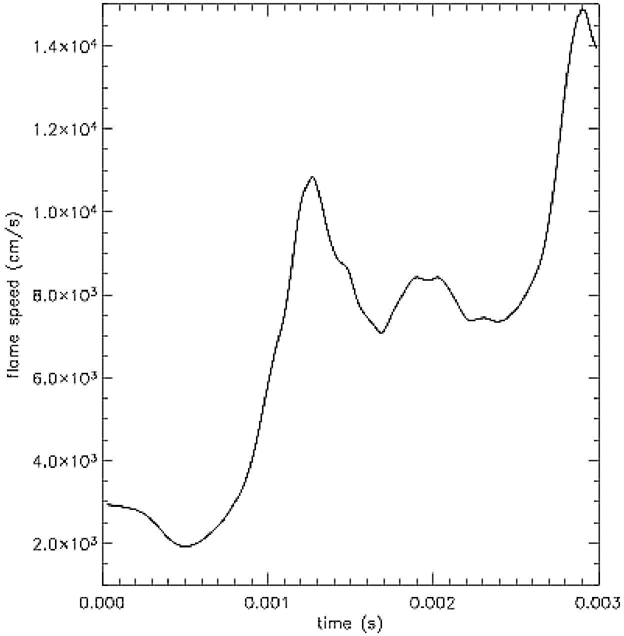


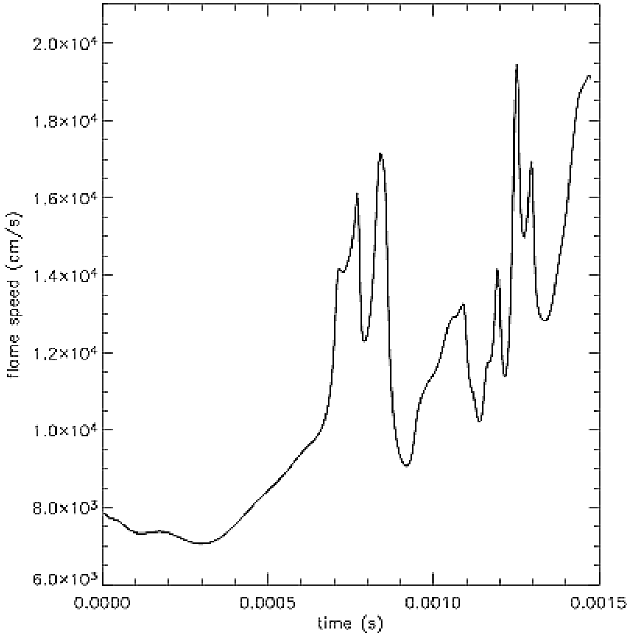
$6.67x10^{6}$ g cm^{-3}



Flame Acceleration







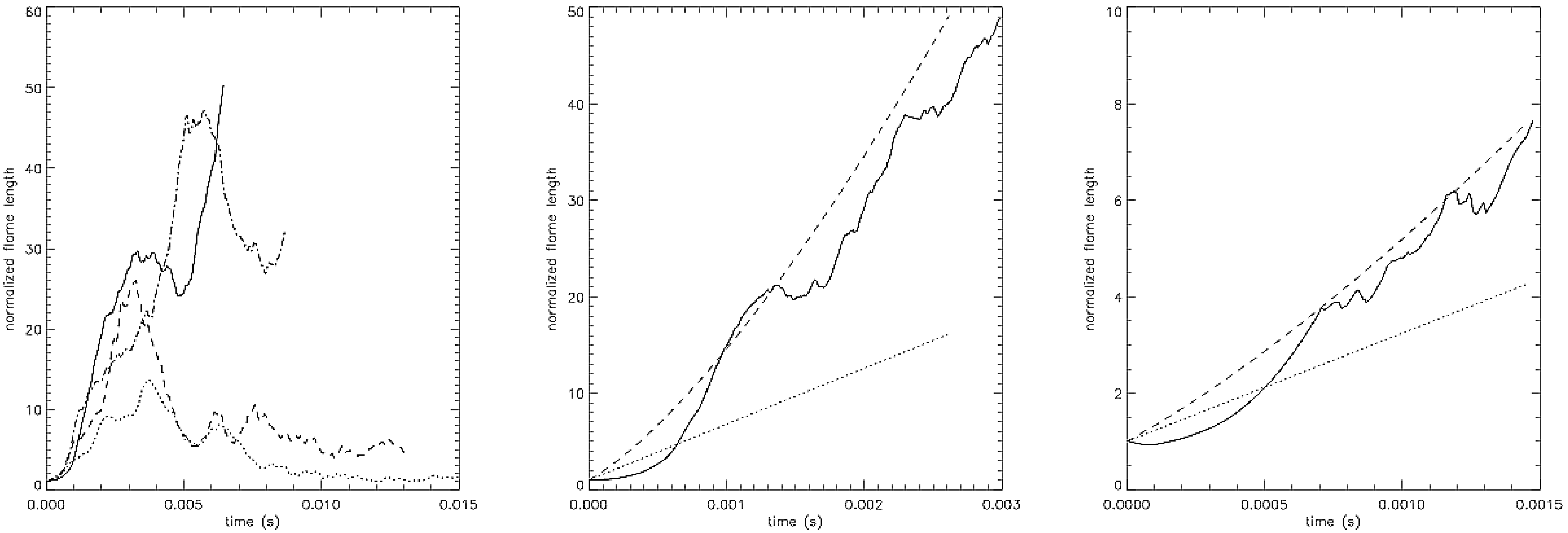
- Flame speed can be computed by looking a carbon consumption rate
- Accelerations up to 6x are obtained
  - Limited only by size of domain

Bell et al. 2003, astro-ph/0401247

Growth of Flame Surface

- Wrinking greatly increases flame length.
  - Increase in flame length > increase in speed ➡ curvature effects are important.
  - Flame length can be fit to a fractal model

$$L = L_0 \left(\frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \right)^{D-1} = L_0 \left(\frac{\alpha g_{\text{eff}}^2 (t - t_0)^2}{4\pi v_{\text{laminar}}^2} \right)^{D-1}$$

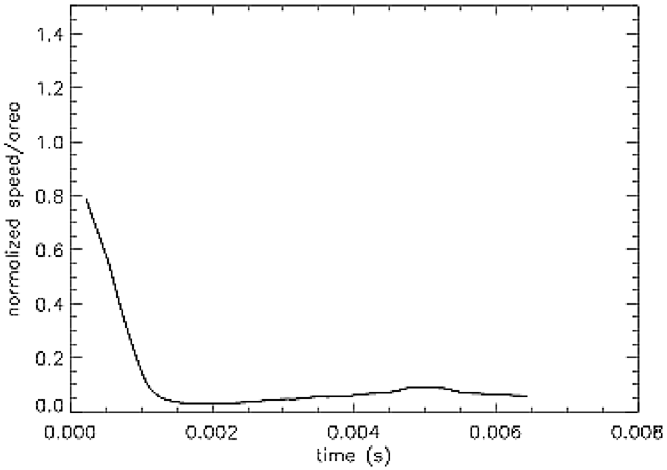


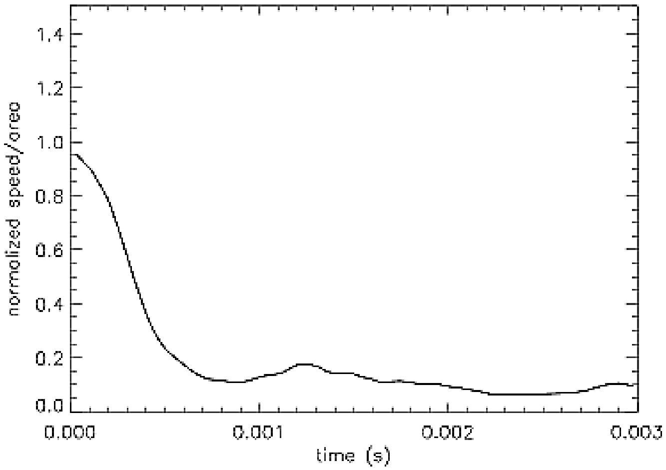
Scaling of Speed with Area

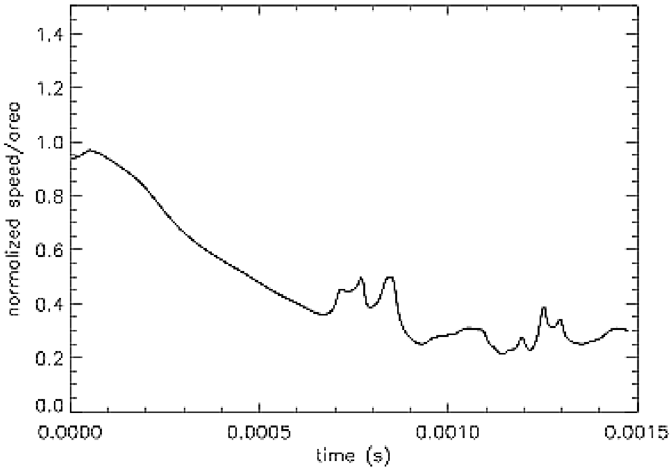
- A simple estimate for the flame speed is that it grows with the surface area

$$v(t) = \frac{A(t)}{A_0} v_0$$

  - This neglects the effects of curvature and strain.







We find a significant departure from v ~ A

Growth of the RT Instability

- RT generated turbulence reaches speeds of > 10^5 cm s^{-1} on scales of 10^3 cm.
  - Peak turbulent kinetic energy grows as t^2.
  - Quickly will dominate over pre-existing turbulence.
  - Non-reactive RT generated turbulent kinetic energy grows faster.
- Mixed region grows slower than Sharp-Wheeler model.
- Extent of reactive region scales with mixed region.
  - There may not be enough time for a DDT.

Implications for Subgrid Models

- Two different mode descriptions are needed:
  - Scaling in the flamelet regime
  - Volume burning in the distributed burning regime
- In the flamelet regime, we can quantify the curvature effects
- Further scaling studies (underway) will assess the validity of the fractal model.
- It seems that as density increases, $v \propto A$ becomes more valid.
- Need to understand the effects of pre-existing turbulence.

Where Do We Go From Here?

- Understanding the behavior of the turbulence requires 3-d simulations (underway)
  - Is the cascade Kolmogorov (usually assumed) or Bulgiano-Obukhov (buoyancy driven)? Niemeyer & Kerstein (1997)
- Formulation of a subgrid model and level set to advect the flame on large scales
  - We can do validation against the DNS flame just presented
- Full star model, including the effects of stratification and expansion.