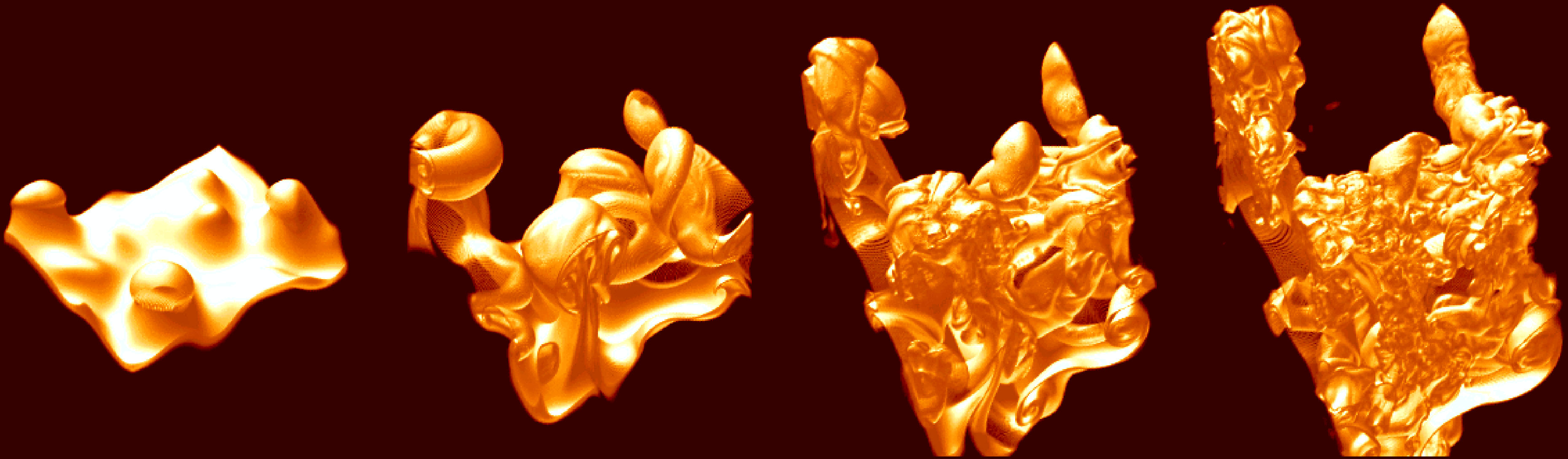
Simulations of Thermonuclear Flames in Type Ia Supernovae



Michael Zingale
(UCSC)

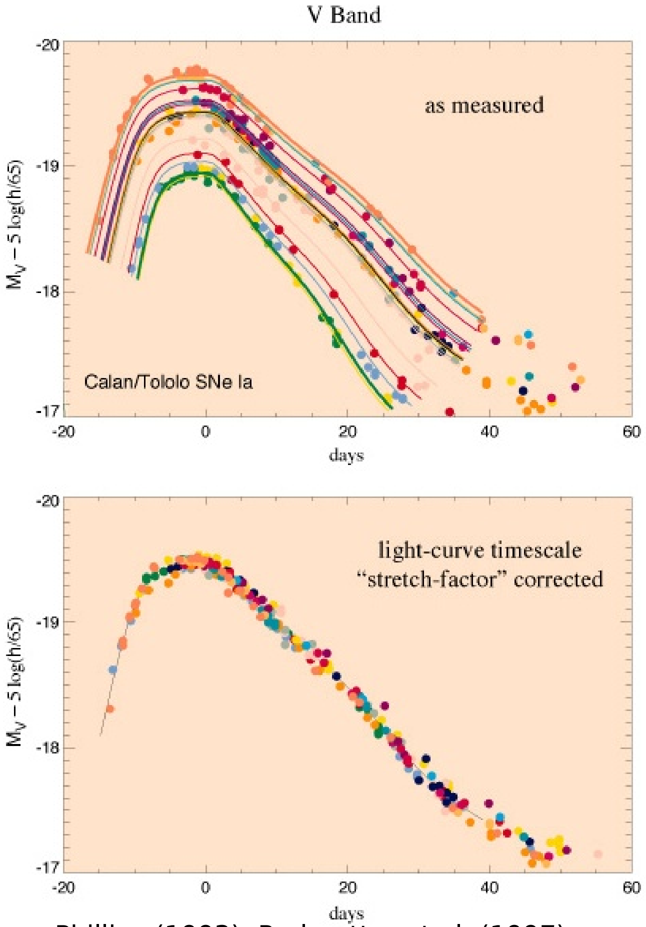
in collaboration with

Ann Almgren, John Bell, Marc Day, Charles Rendleman (LBL), Stan Woosley (UCSC)

Type Ia Supernovae

- Bright as host galaxy, L ~10^{43} erg s^{-1}
- Large amounts of ^{56}Ni produced
  - Radioactivity powers the lightcurve

SN 1994D (High-Z SN Search team)



Phillips (1993), Perlmutter et al. (1997)

- Lightcurve is robust
  - Variations can be corrected for via a single parameter function.
- Thermonuclear explosion of C/O white dwarf.
  - Must begin as a deflagration
  - Considerable acceleration required

Explosion Requirements

- Flame must accelerate to ~ 1/3 c_s.
- Must produce intermediate mass elements (Si, S, Ar, Ca).
- Produces ~ 0.6 $M_\odot$ ^{56}Ni.
- How does the flame accelerate?
  - Flame instabilities (Landau-Darrieus, Rayleigh-Taylor)
  - Interaction with turbulence.

Increase surface area ⇒ increase flame speed.

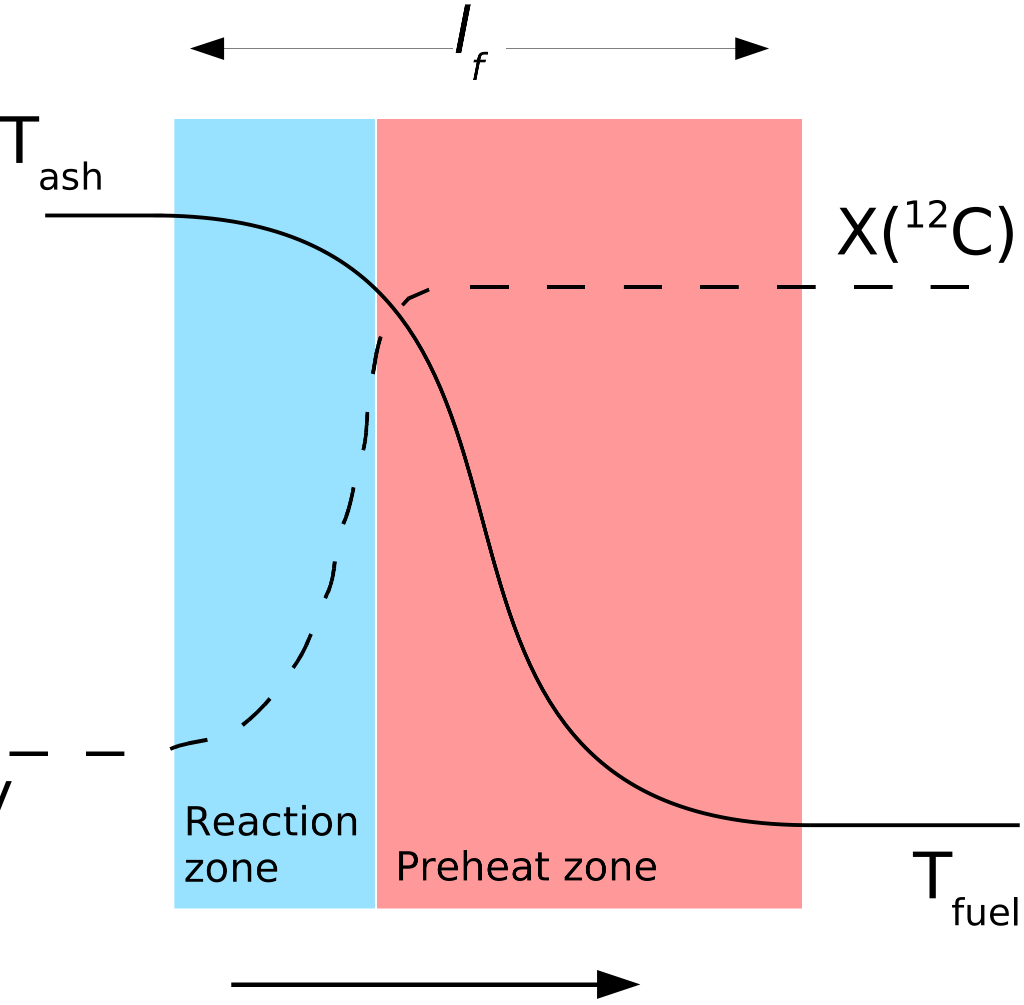
Type Ia Supernovae Theory

- Ra ~ 10^{25} (buoyancy to diffusion forces)
  - Nature of convection is not well known in this regime.
- Re ~ 10^{14} (inertial to viscous forces)
- Pr ~ 10^{-4} (momentum transport to heat conduction)
  - Viscosity effects are unimportant.
- Le ~ 10^{7} (energy transport to mass transport)
  - Mass diffusion can be neglected.
  - Large departure from typical Le ~ 1 terrestrial flames.

Wunsch, Woosley, Kuhlen (2003)

Flames

- Begins as a deflagration
  - Subsonic burning front
    - Pressure is constant
    - Density drops in the ash region.
  - Thermal diffusion transports the heat
- Laminar speed too slow (Timmes and Woosley 1992)
  - Must accelerate considerably at low densities.
  - May transition to detonation

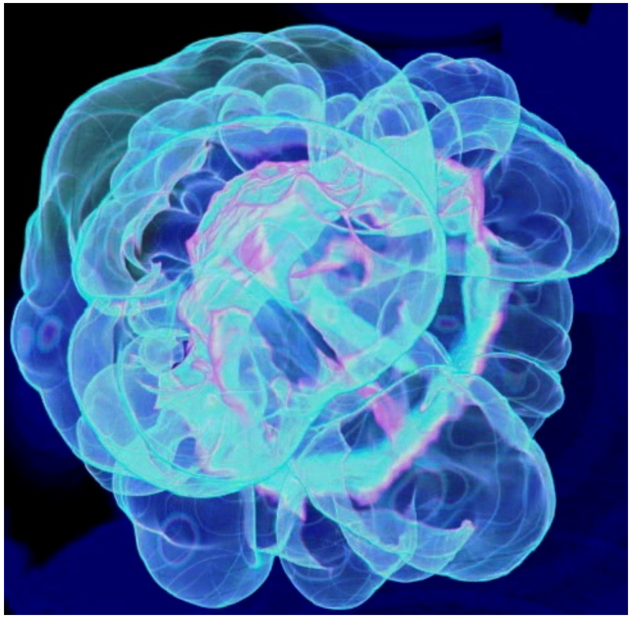


SNe Ia Unstable Flames

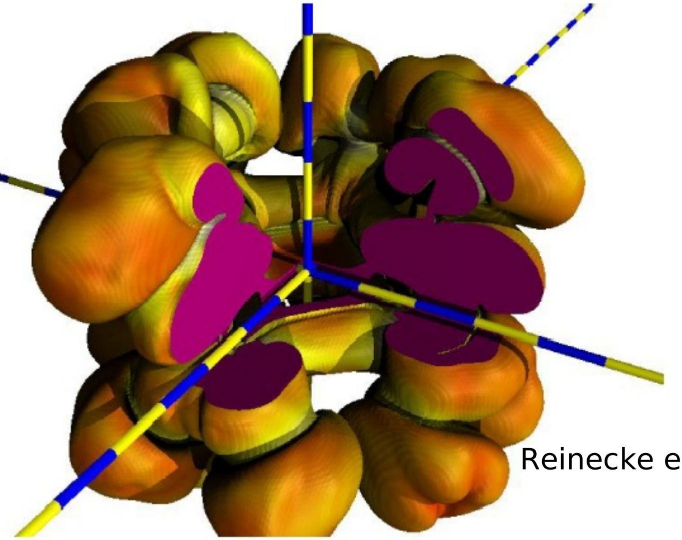
- Explosion begins as a flame in the interior of the white dwarf.
  - ~ 100 years of convection preceed ignition
  - subsonic propagation allows the star to expand.
- Hot ash is less dense than the cool fuel.
- Subjected to numerous instabilities.

Large Scale Simulations

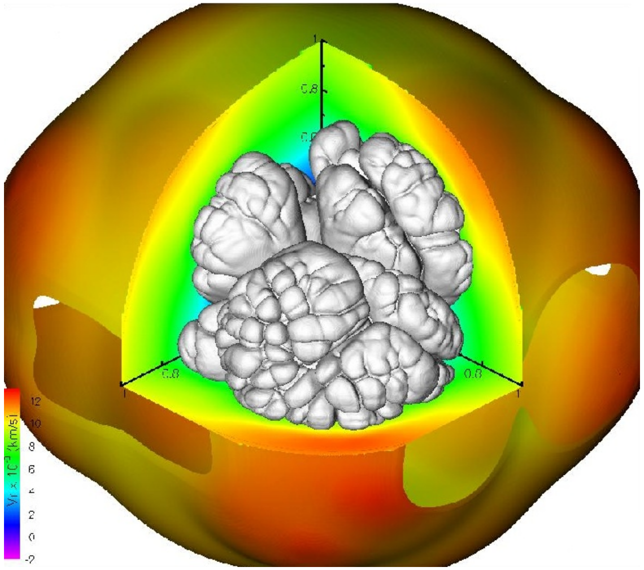
- Instabilities are the dominant acceleration mechanism.
- Pure deflagrations can unbind the star.



Calder et al. (2004)



Reinecke et al. (2003)

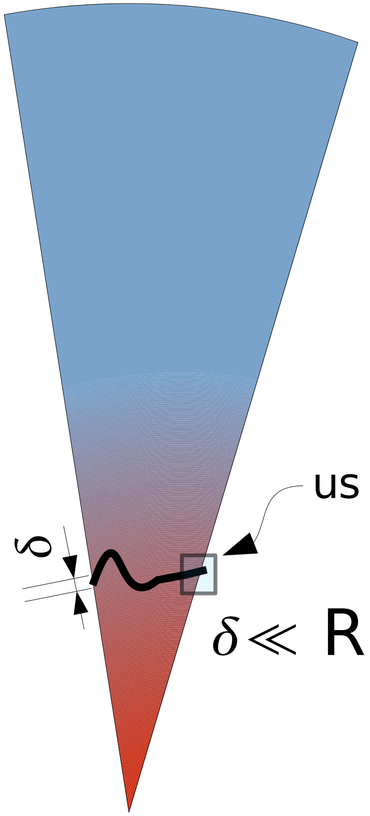


Gamezo et al. (2003)

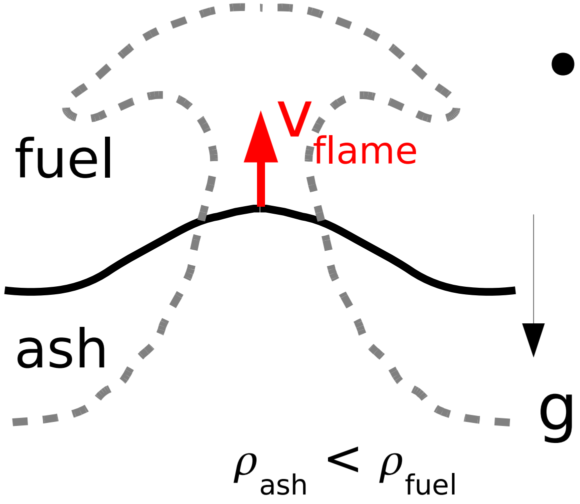
- Some flame model is required.
  - Stellar scale ~ 10^8 cm
  - Flame width ~ 10^{-5} – 10 cm

Bottom-Up Approach

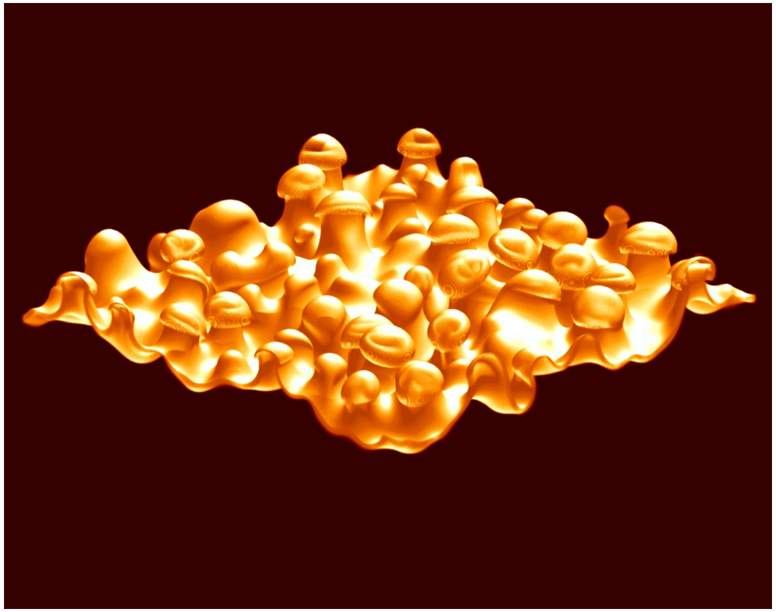
- Simulations cannot resolve the star and the flame.
  - Modern adaptive mesh methods/ massively parallel computers can handle 3 orders of magnitude
- We resolve the structure of the flame and work up to large scales
  - Parameter free.
  - Resolved calculations can be used to validate flame models.
  - Sometimes we will need a supergrid model
- Look for scaling relations that will act as subgrid models.



Reactive Rayleigh-Taylor Instability



- Rayleigh-Taylor
  - Buoyancy driven instability.
  - Large amounts of surface area generated.



Zingale et al. (2005)

- Sharp-Wheeler model predicts mixed region growth:

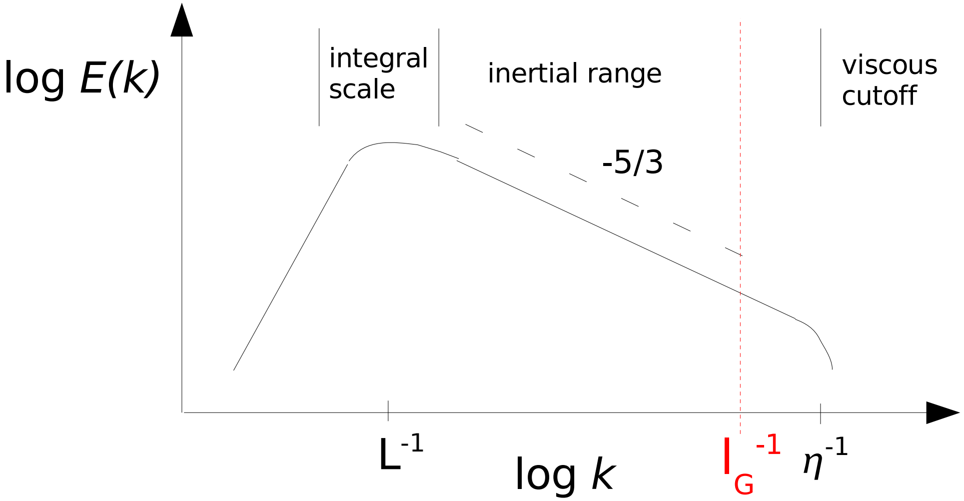
$$h = \alpha A g t^2$$

- Reactions set a small scale cutoff to the growth of the instability:

$$\lambda_{\text{fp}} = 4\pi \frac{v_{\text{laminar}}^2}{g_{\text{eff}}}$$

Turbulence

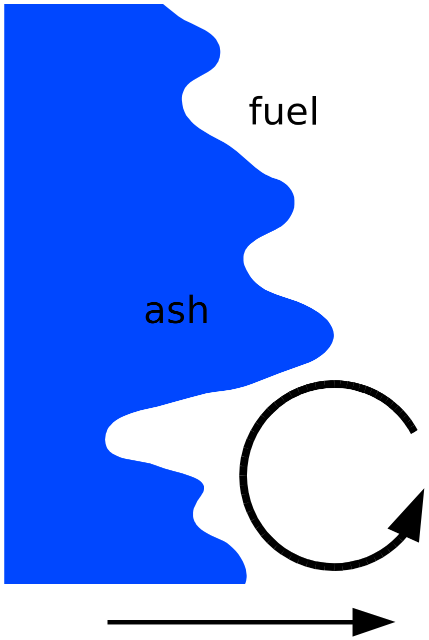
- Kinetic energy cascade over a range of length scales
  - Integral scale, L: bulk of kinetic energy exists
  - Kolmogorov scale, η: inertial and viscous effects balance
  - Gibson scale, l_G: eddy turns over before burning away.



adapted from Peters (2000)

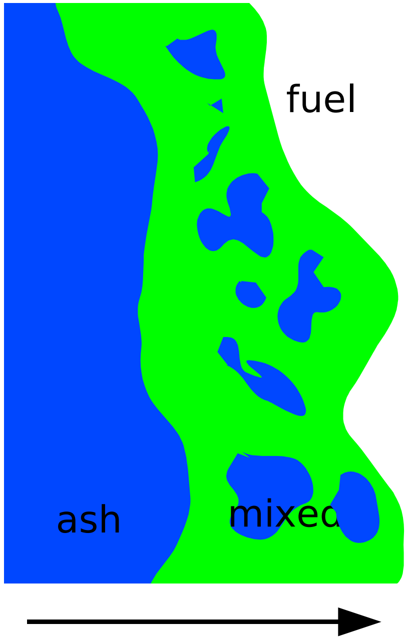
- Size of l_G in comparison to flame width determines the flame regime.

Transition to Distributed Burning



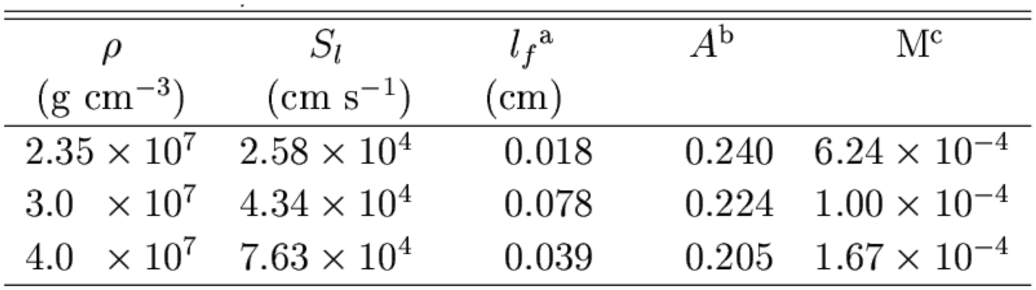
- Flame begins as flamelet
  - Flame is a continuous surface
  - Turbulence serves solely to wrinkle the flame, increasing the area

- Transition to distributed burning regime is proposed at ~10^7 g cm^{-3}
  - Mixed region of fuel + ash develops
  - May be possible to quench the flame
  - Possible transition to detonation

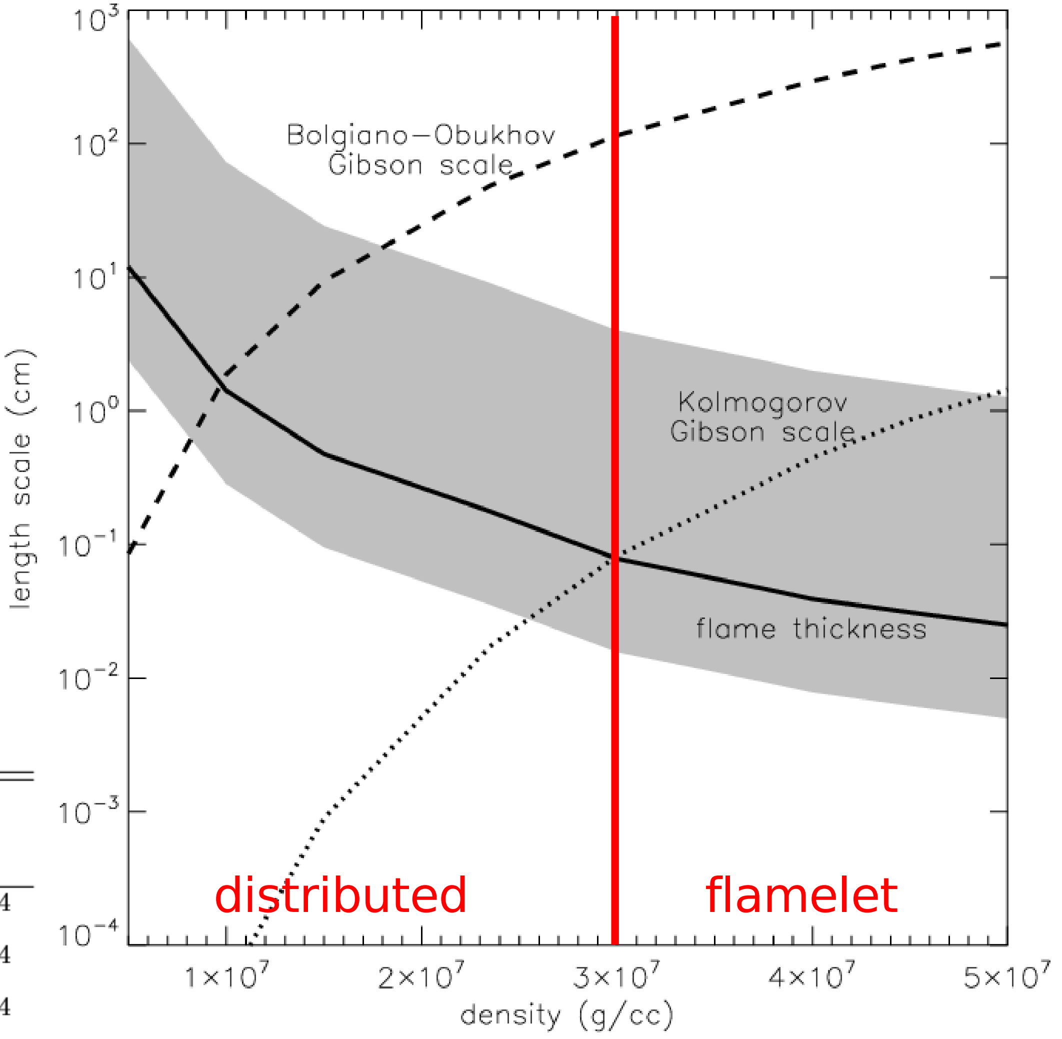


Low Density Flame Properties

- $l_G = l_f$ at $3\text{x}10^7$ g cm^{-3}
  - Transition to distributed burning expected
    (Niemeyer & Woosley 1997)
  - We need to resolve both scales
  - Flames are very low Mach number



ρ (g cm^{-3})	S_l (cm s^{-1})	l_f[a] (cm)	A[b]	M[c]
2.35×10^7	2.58×10^4	0.018	0.240	6.24×10^{-4}
3.0×10^7	4.34×10^4	0.078	0.224	1.00×10^{-4}
4.0×10^7	7.63×10^4	0.039	0.205	1.67×10^{-4}



Low Mach Number Hydrodynamics

(Bell et al. 2004 JCP 195, 677)

- Low Mach number formulation projects out the compressible components.
  - Pressure decomposed into thermodynamic and dynamic components.

$$p(x,t) = p_0(t) + Mp_1(t) + M^2\pi(x,t)$$

  - Elliptic constraint provided by thermodynamics.

$$0 \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial \rho}\frac{D\rho}{Dt} + \frac{\partial p}{\partial T}\frac{DT}{Dt} + \sum_k \frac{\partial p}{\partial X_k}\frac{DX_k}{Dt}$$

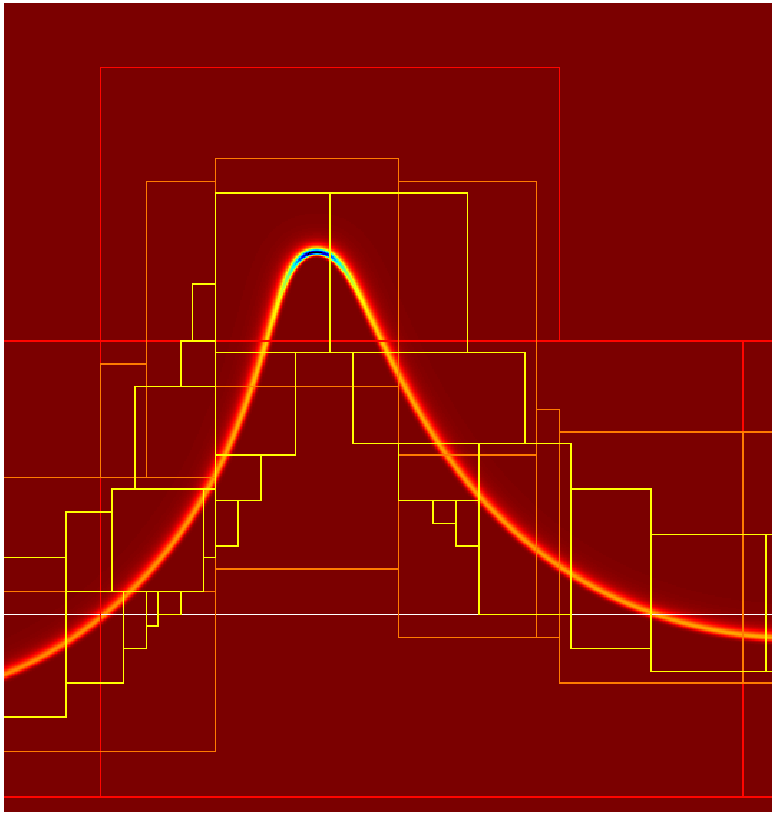
$$\nabla \cdot U = \frac{1}{\rho \frac{\partial p}{\partial \rho}}\left(\frac{\partial p}{\partial T}\frac{DT}{Dt} + \sum_k \frac{\partial p}{\partial X_k}\frac{DX_k}{Dt}\right)$$

  - Advection/Projection/Reaction formulation solves system.
  - Timestep limited by |v| and not |v| + c.

Simulation Method

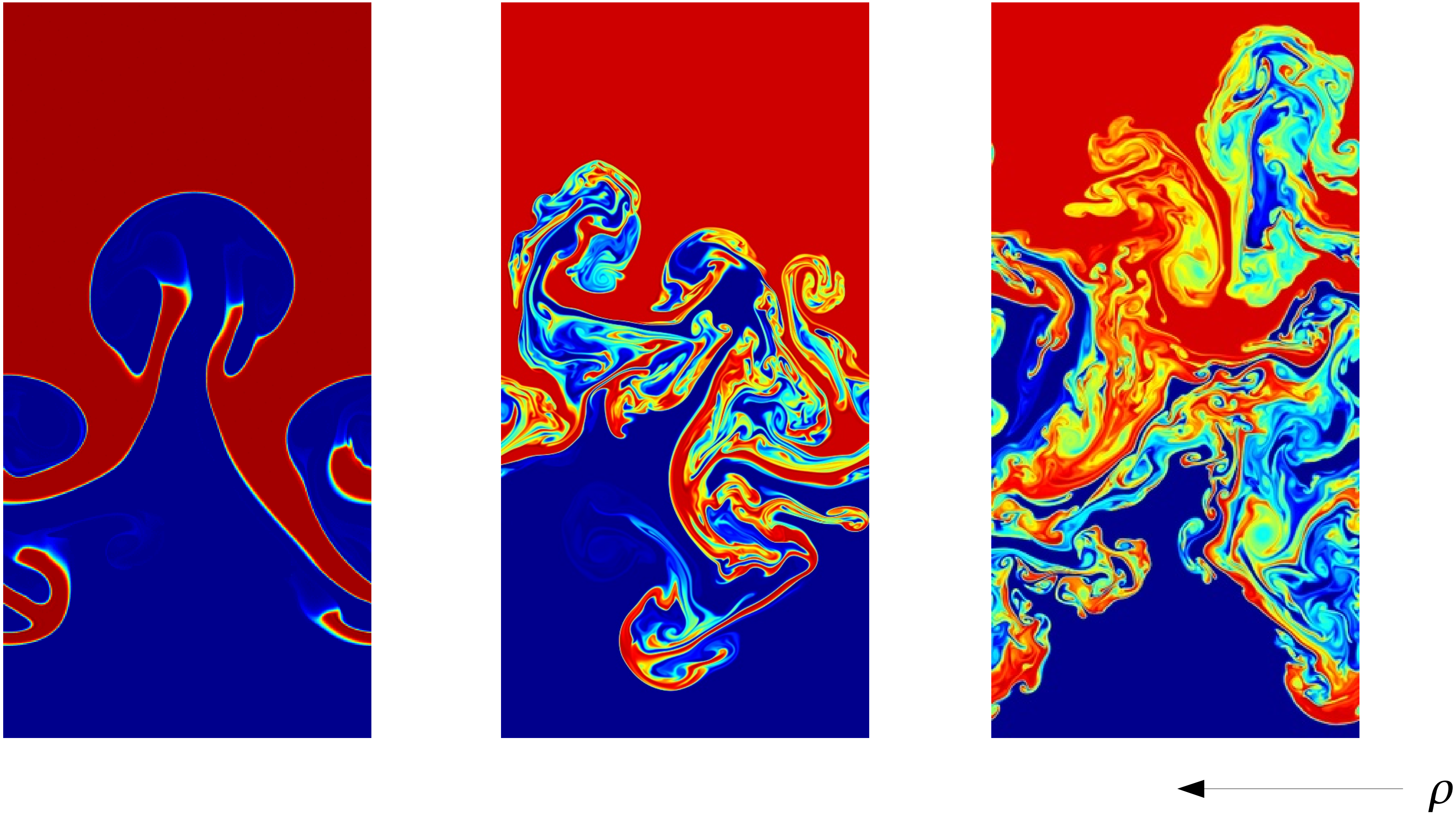
(Bell et al. 2004 JCP 195, 677)

- Low Mach number hydrodynamics.
  - Advection/projection/reaction
  - Block structured adaptive mesh
  - Timestep restricted by |v| not |v| + c
  - Degenerate/Relativistic EOS used.
  - Single step ^{12}C+^{12}C rate
- Initialized by mapping 1-d steady-state laminar flame onto grid.
  - 5-10 zones inside thermal width.



Transition to Distributed Burning

(Bell et al. 2004, ApJ, 608, 883)

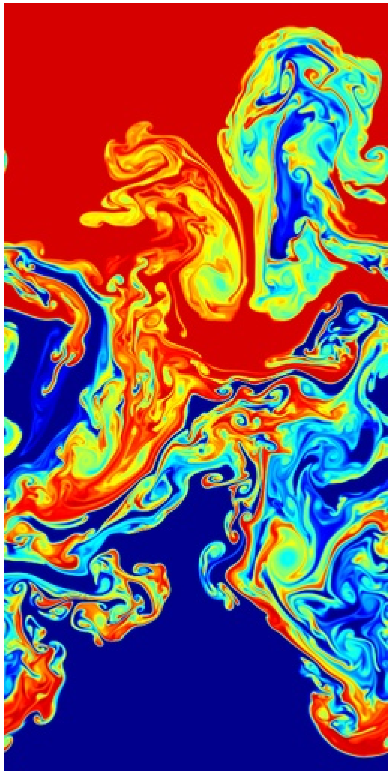


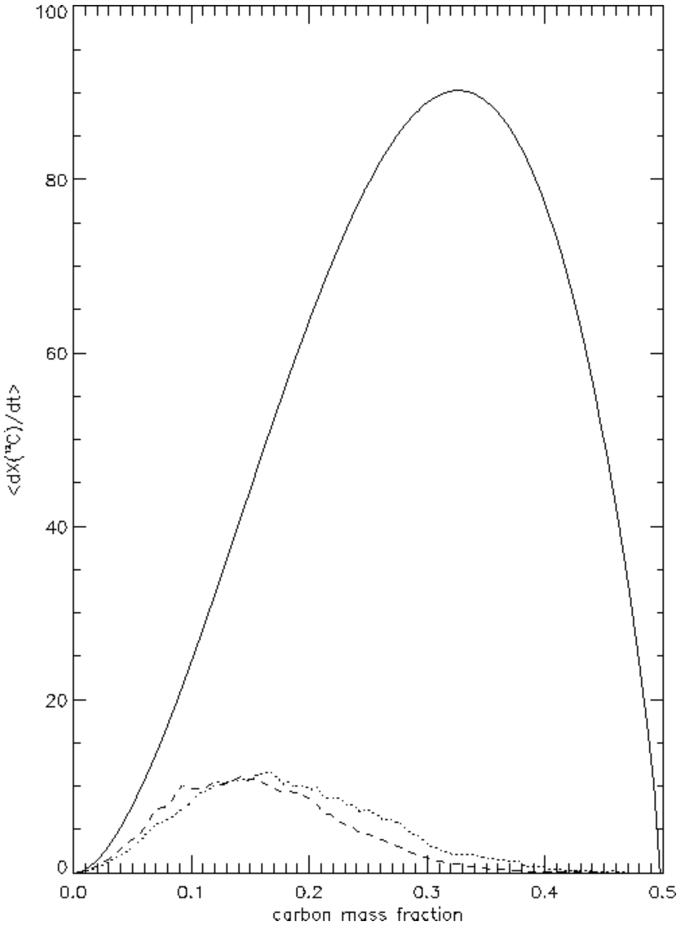
- As ρ decreases, RT dominates over burning.
- At low ρ, flame width is set by mixing scale.

Deflagration-Detonation Transition

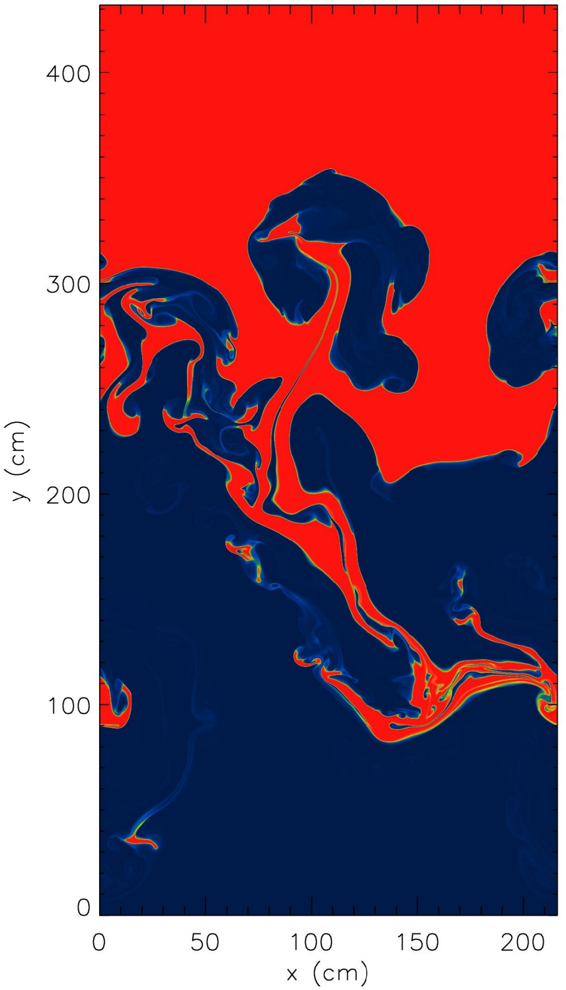
(Bell et al. 2004, ApJ, 608, 883)

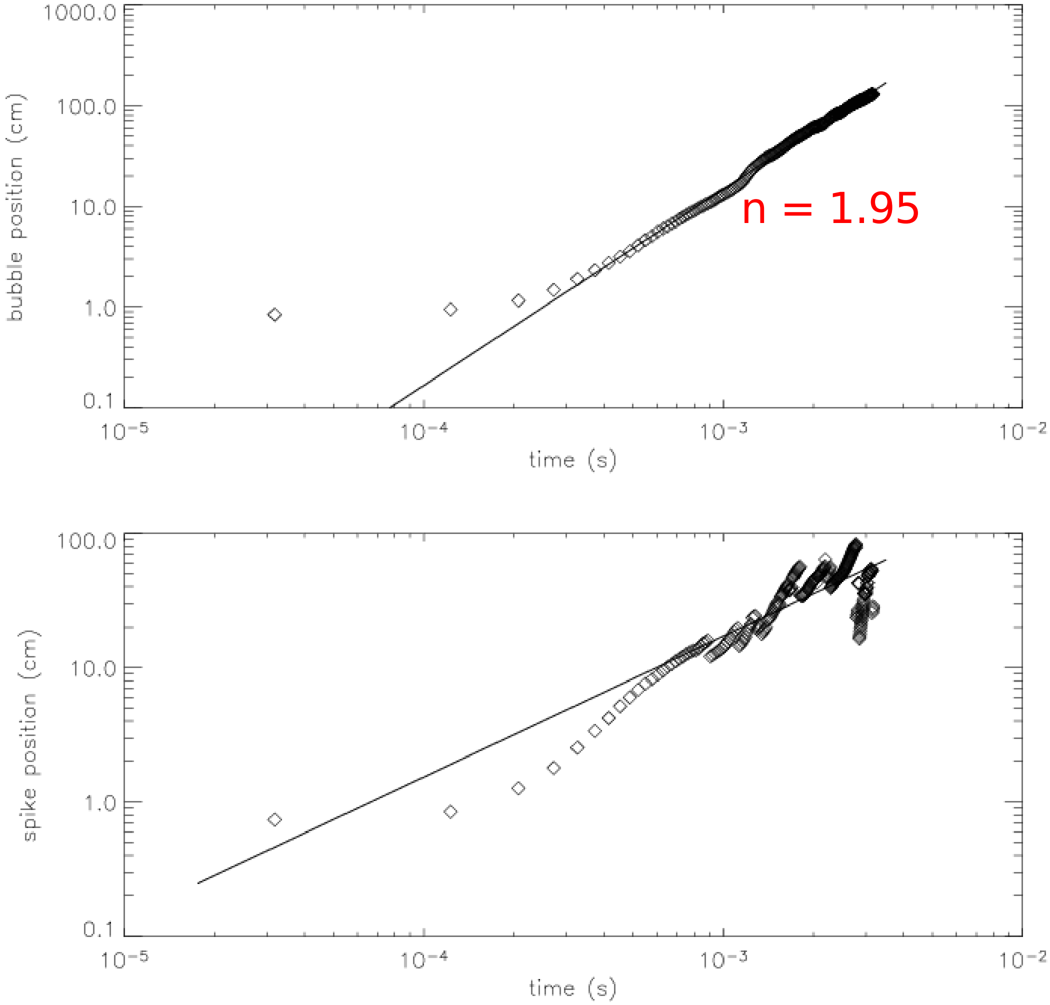
- In the distributed regime, fuel burns at $X_{^{12}C} \sim 0.15$
  - Detonation matchhead is larger than the star.
  - Localized transition to detonation is unlikely.





Growth of the Mixed Region





- Mixed region does not grow as Sharp-Wheeler
  - Interface between mixed/ash burns away
  - $\alpha = 0.047$

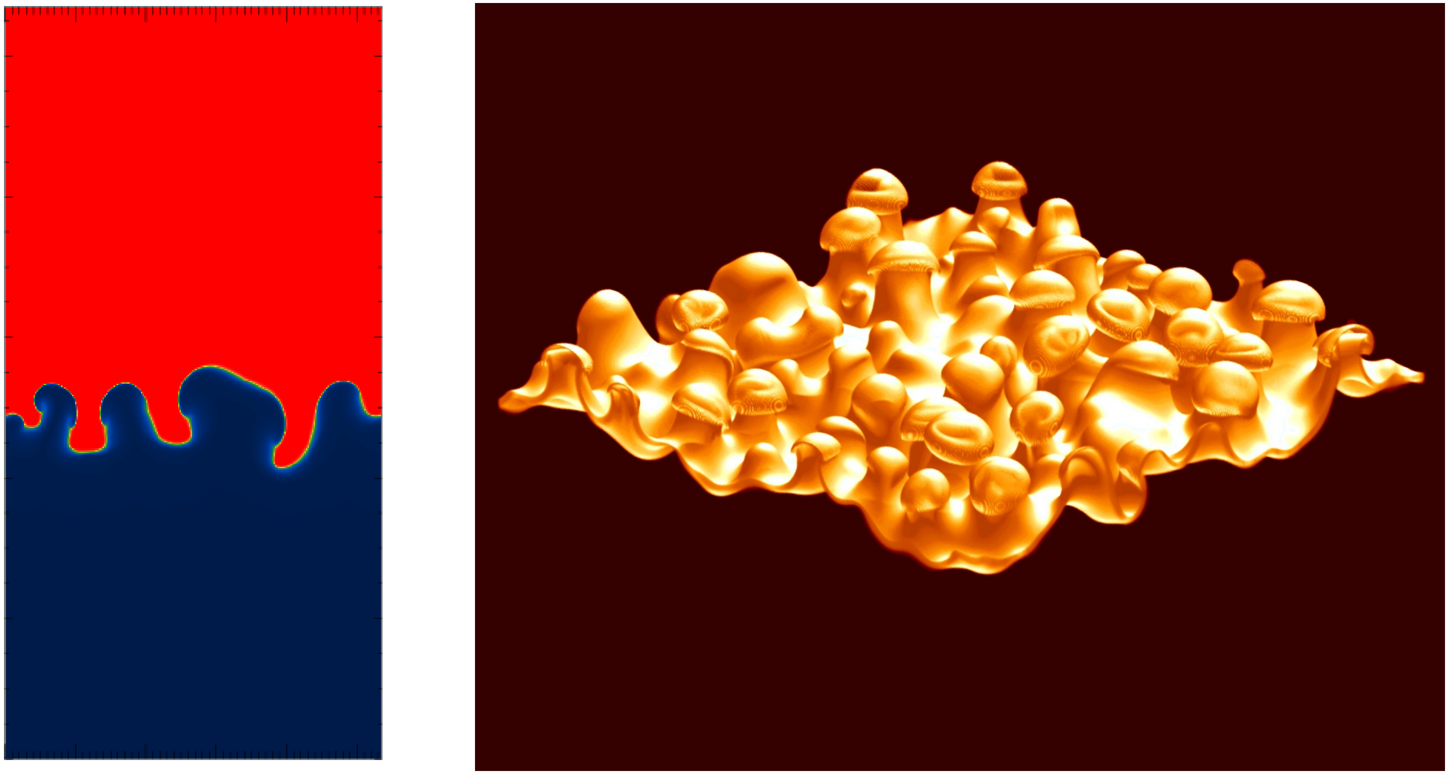
2-D Reactive RT: Transition to Distributed Burning Summary

- Accelerations to several times the laminar speed
  - Limited only by the size of the domain.
- Transition to distributed burning occurs at density of 10^7 g cm^{-3}
- Growth of reactive region scales with mixed region
  - There does not appear to be enough time for a localized transition to detonation.
- Curvature/strain effects become quite important near the transition.

3-D Reactive RT

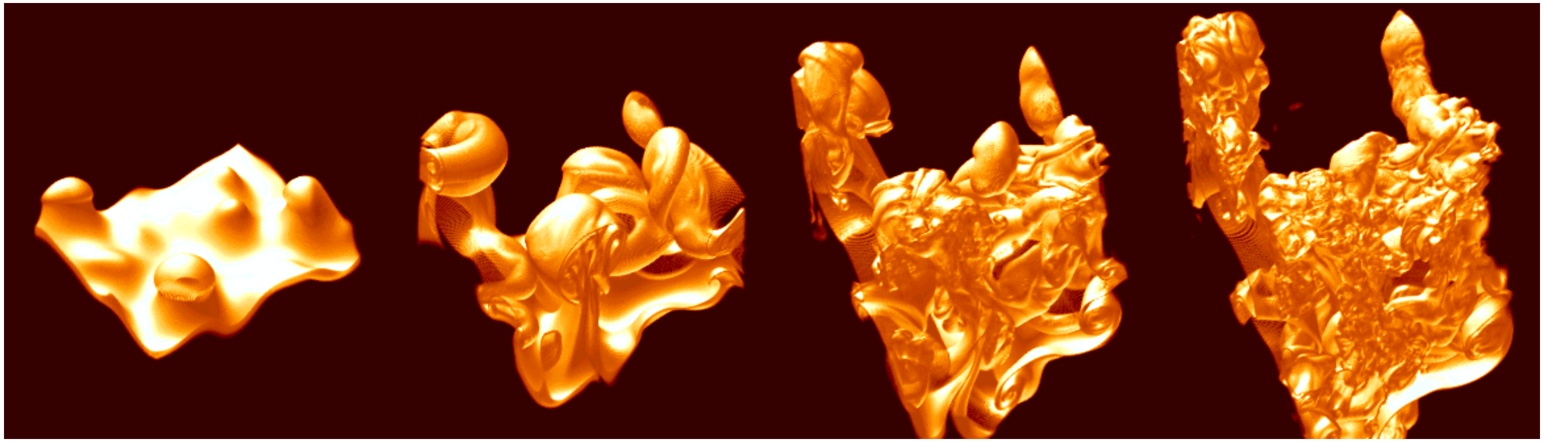
(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

- 3-D analogue of 2-D runs previously studied
  - 512 x 512 x 1024 effective zones
  - Surface to volume is greater
  - Fire-polished RT dominates the early evolution.



3-D RT: Transition to Turbulence

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

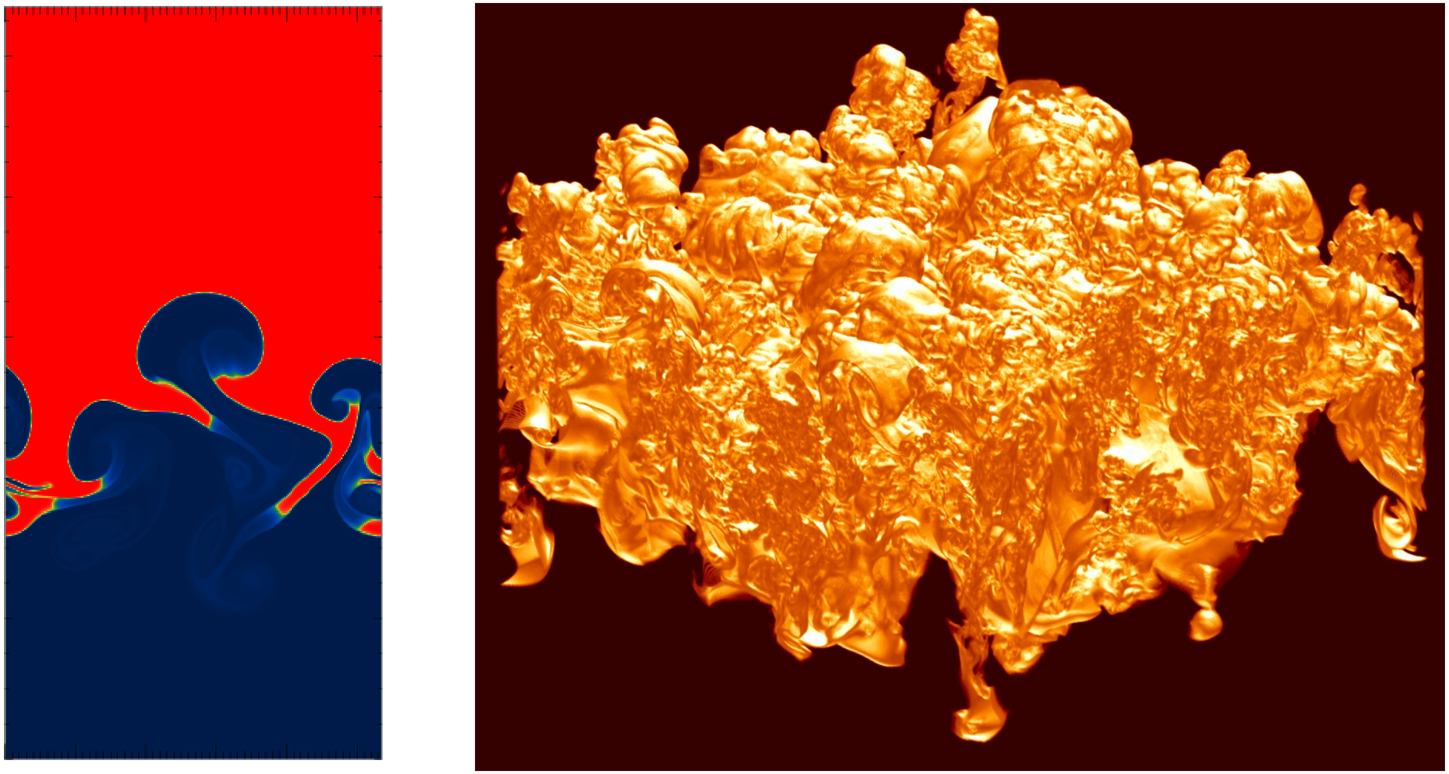


- Turbulence generated on the large scales cascades down, and causes wrinkling on scales smaller than λ_{fp} .
  - This cannot happen in 2-D.

3-D Reactive RT

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

- At late times, a fully turbulent flame propagates
  - No analogy to the 2-D case.
  - Evolution now dominated by turbulence, not Rayleigh-Taylor.

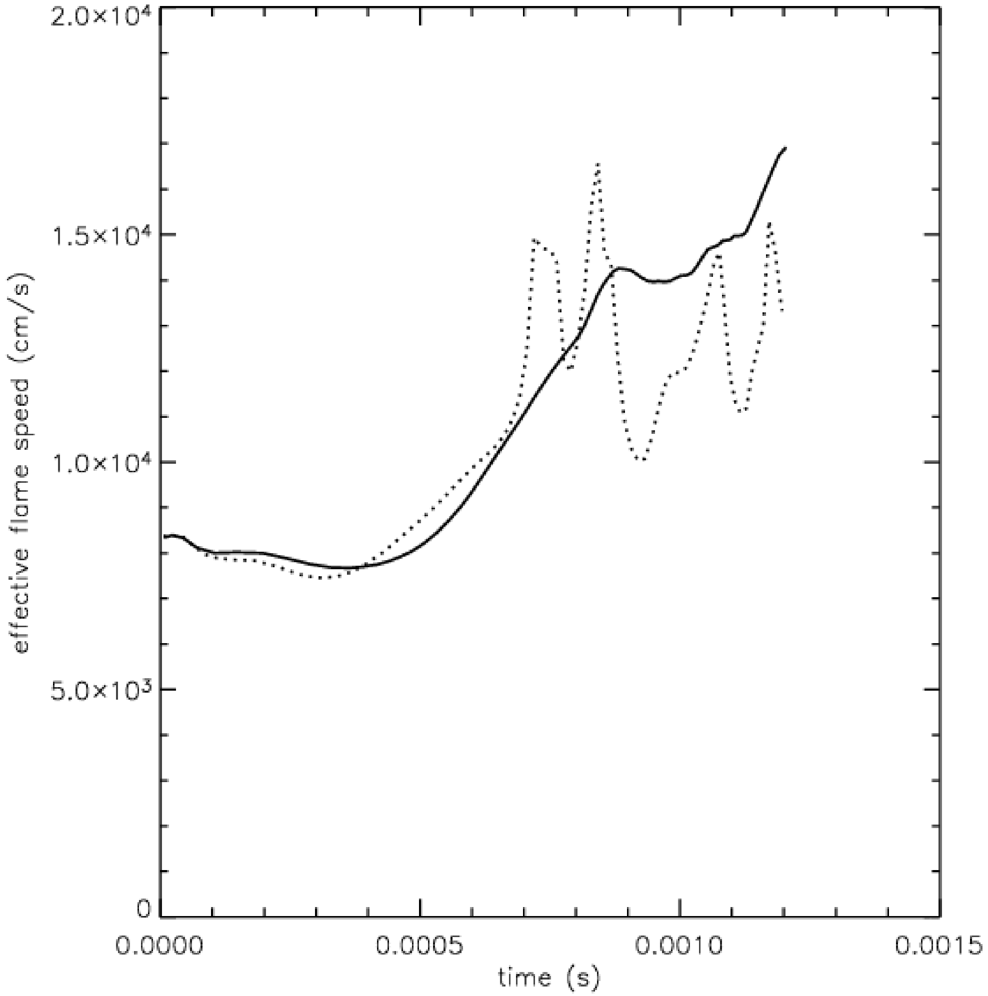


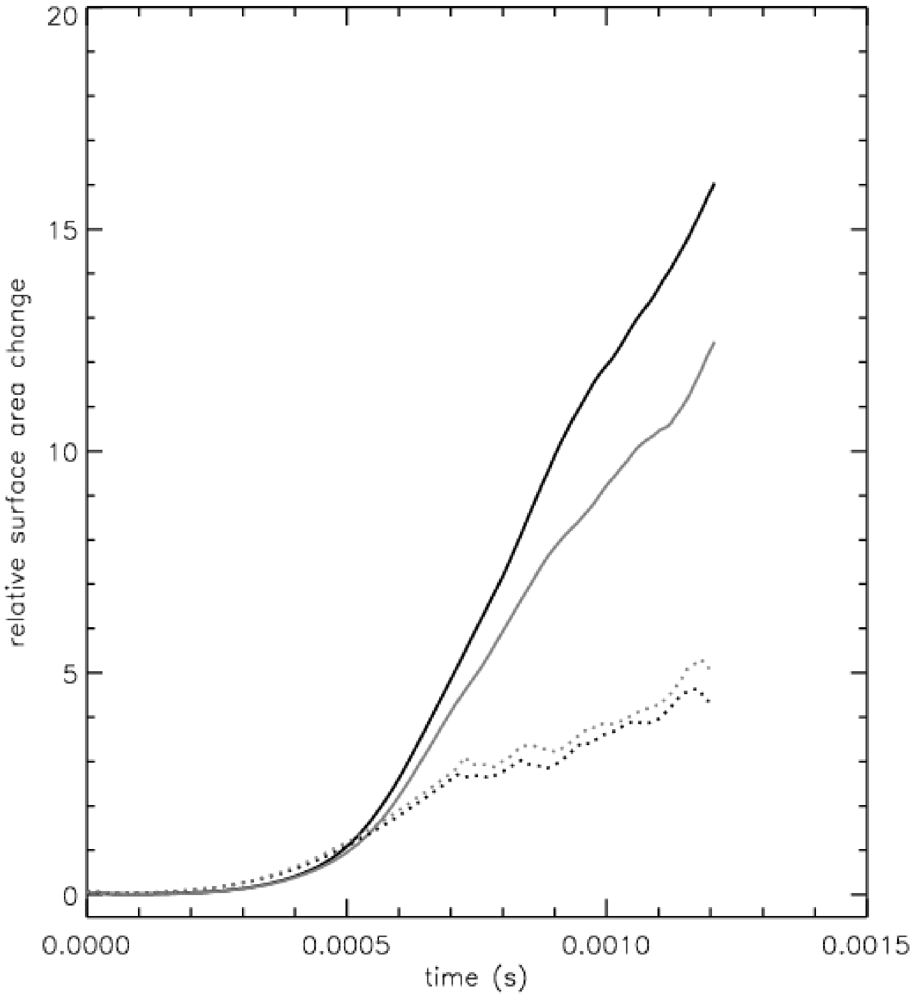
Animation of Rayleigh-Taylor Flame

3-D Reactive RT

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

- Late time acceleration in 3-d due to interaction with flame generated turbulence





Power Spectrum

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

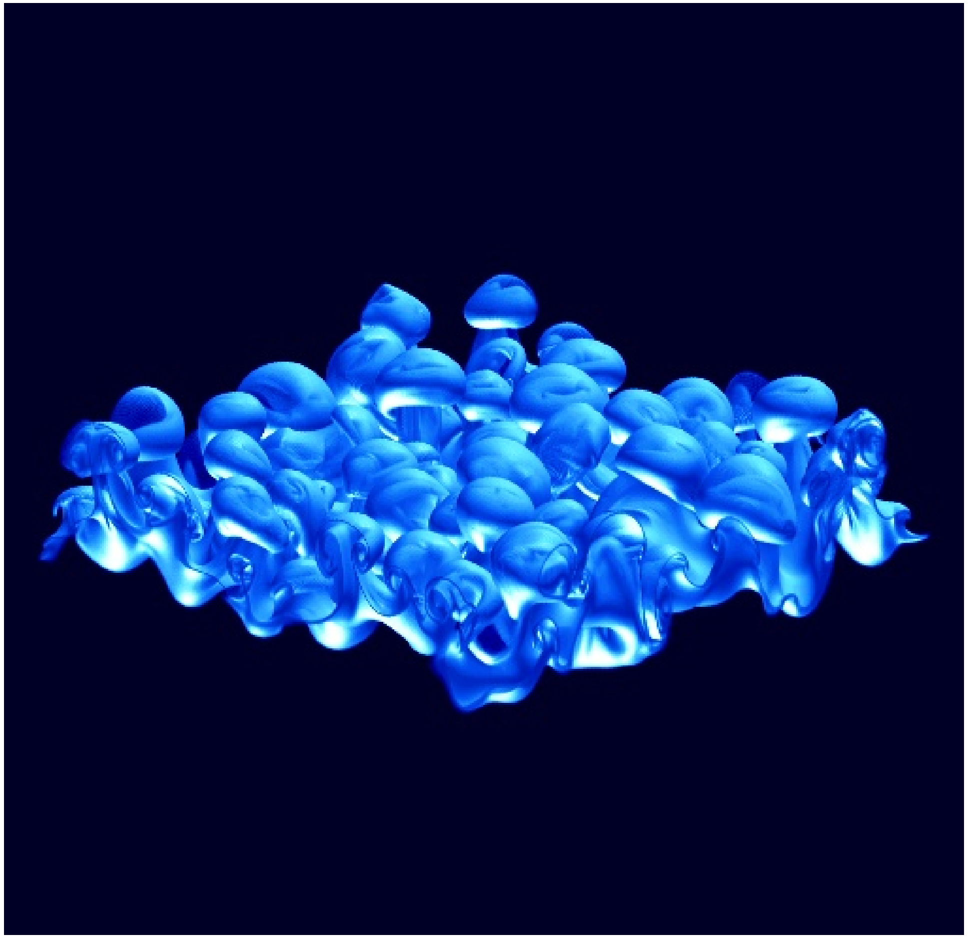
- Power spectrum can be used to determine the nature of the turbulence
  - Our domain is not periodic in all directions (inflow and outflow boundaries)
  - Velocity field is decomposed into divergence free part + effects of boundaries and compression

$$\mathbf{u} = \mathbf{u}_d + \nabla\phi + \nabla\psi$$

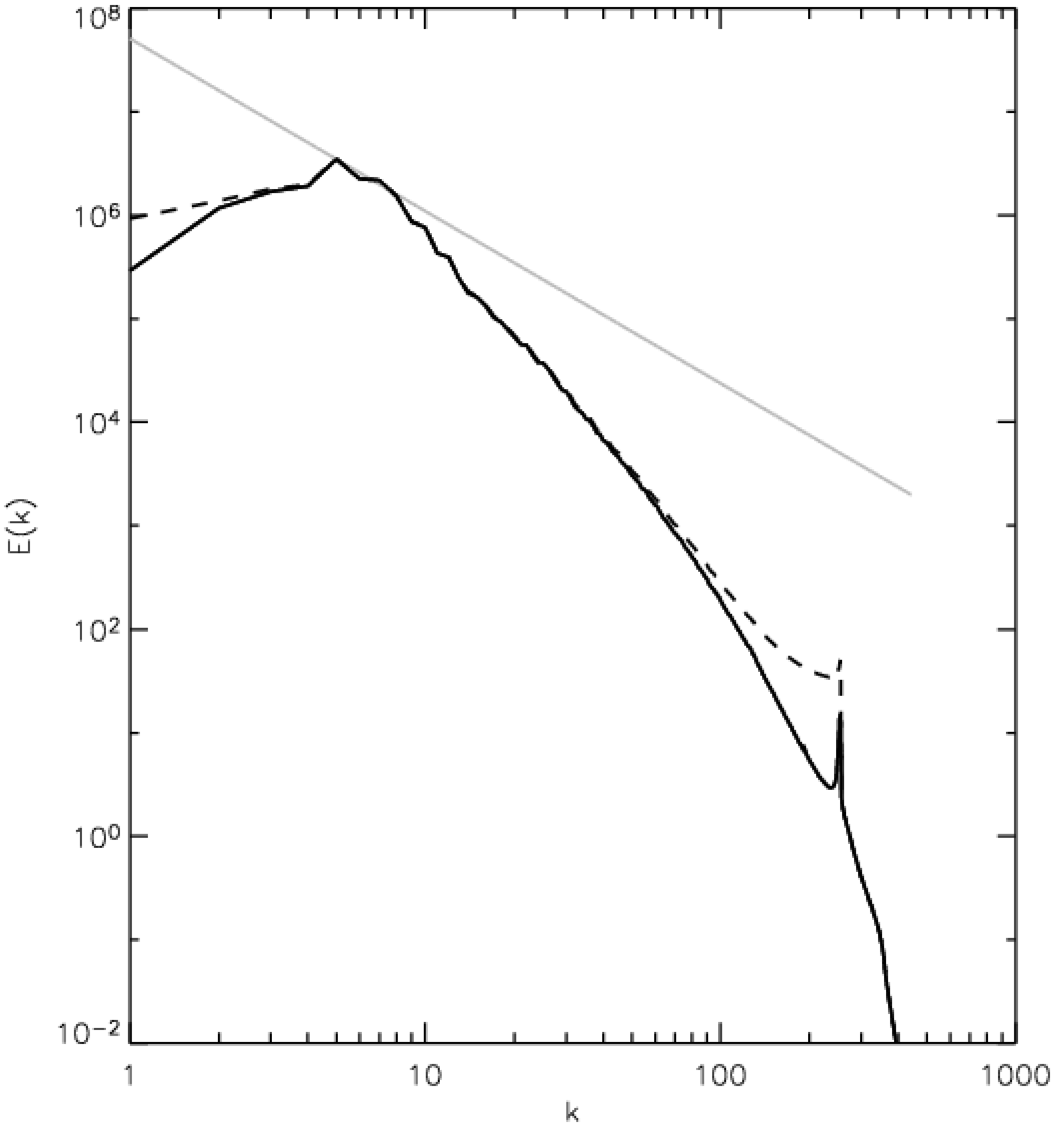
  - Divergence free part is projected out.
  - FFT is performed on divergence free field

Transition to Turbulence

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

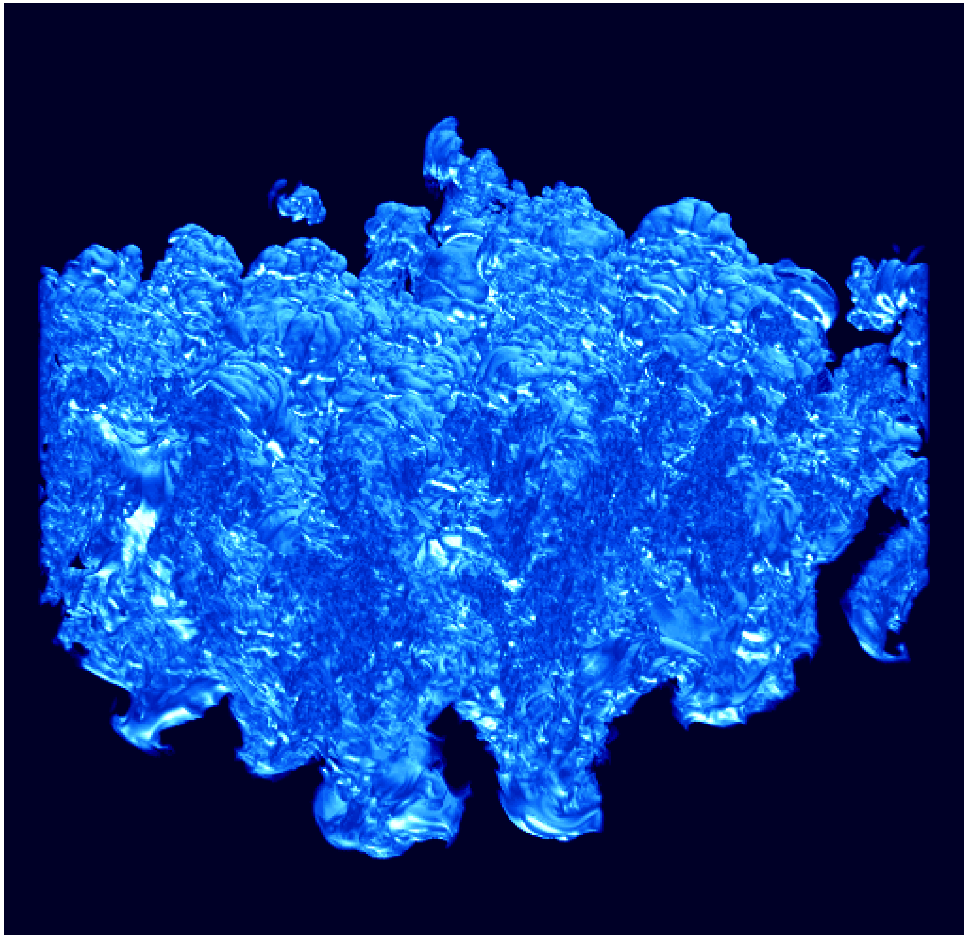


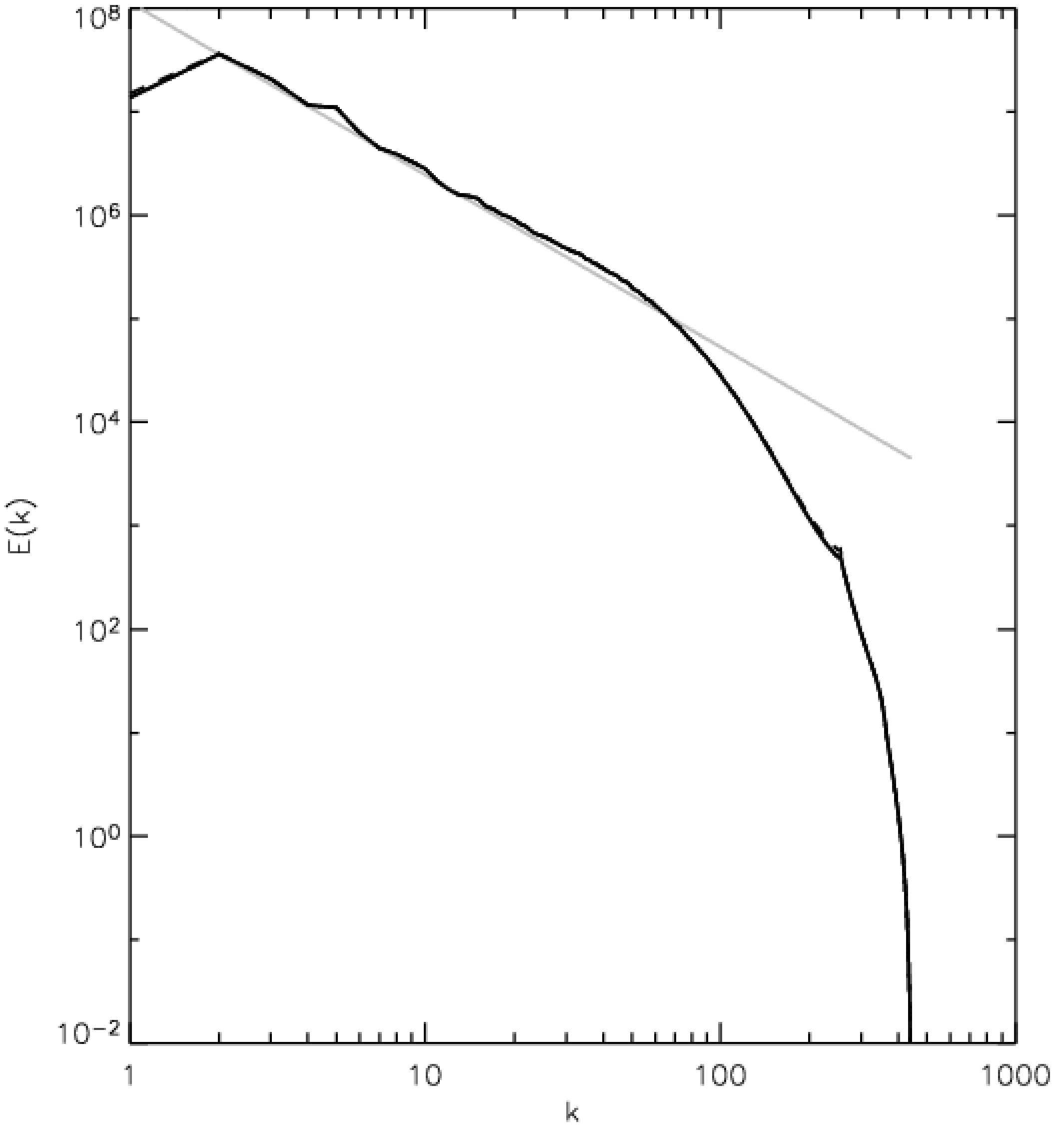
t = 6.62 x 10^{-4} s



Transition to Turbulence

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

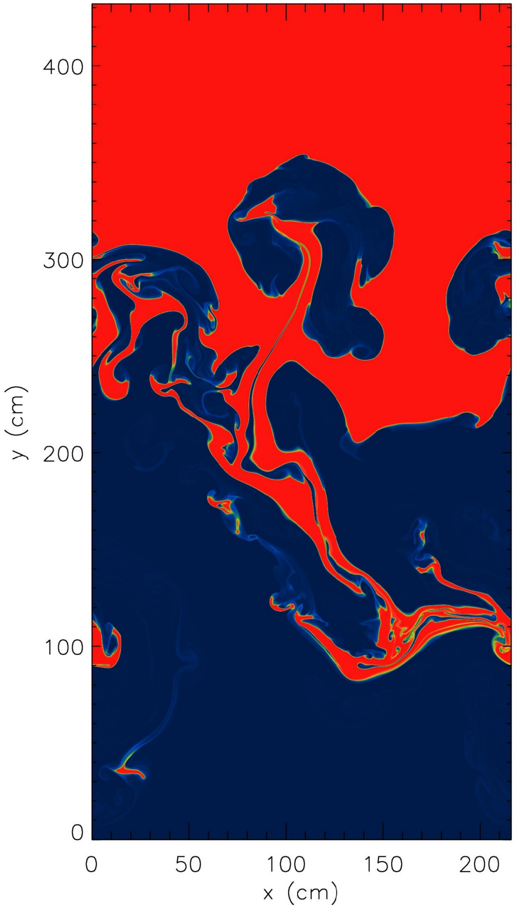


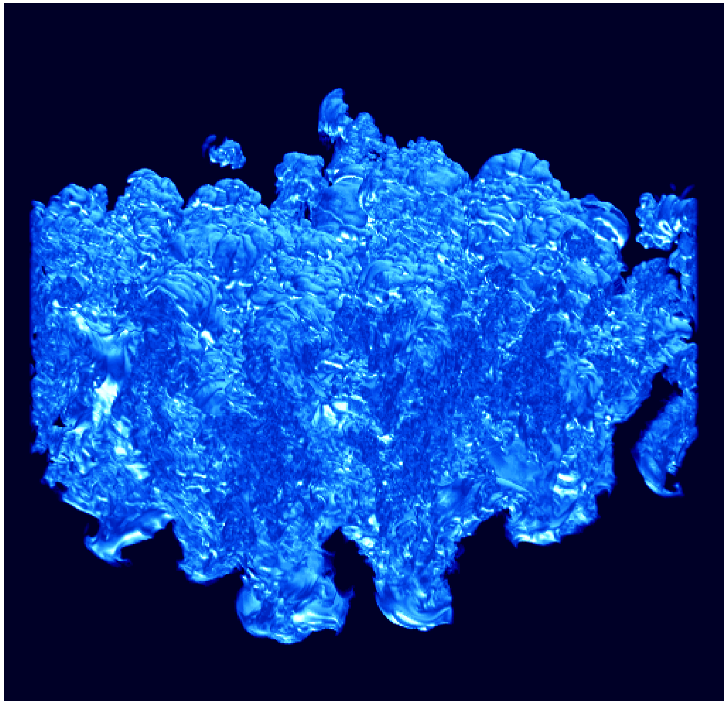


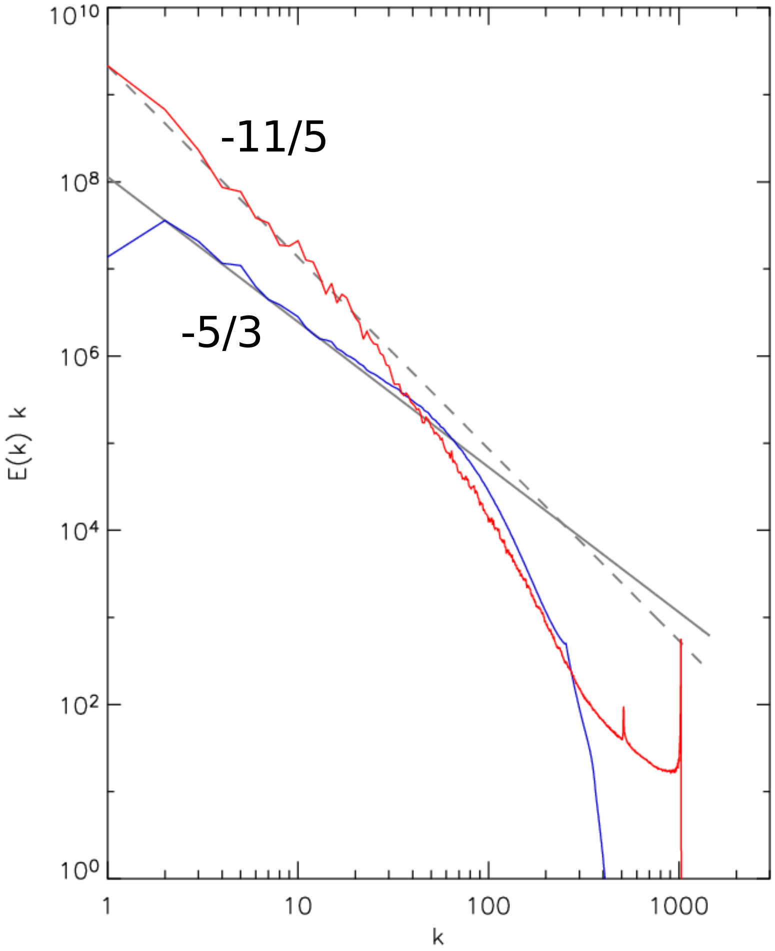
t = 1.16 x 10^{-3} s

Differences Between 2- and 3-D

(Zingale et al. 2005, J Phys Conf Series, 16, 405)





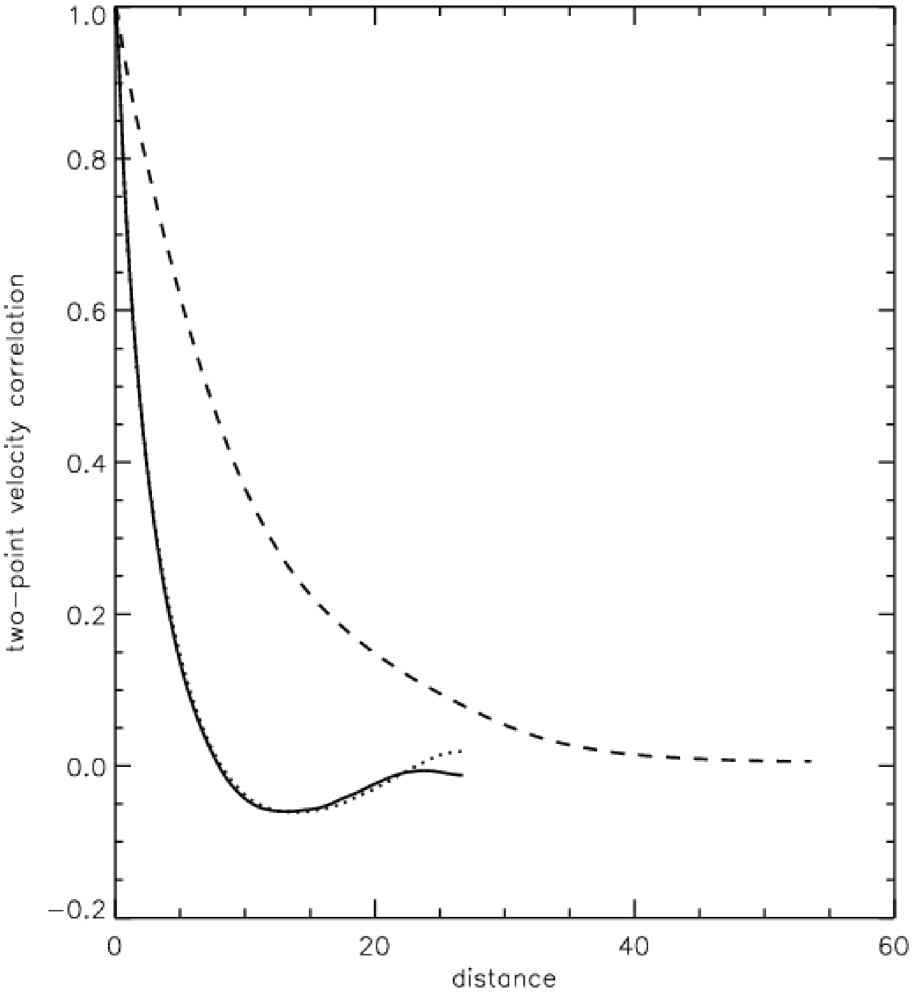


- The turbulent cascade is different in 2- and 3-D.
  - Kolmogorov scaling is only seen in 3-D
  - Flame calculations need to be 3-D

Integral Scale

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

$$l_t^{(x)} = \frac{1}{\int_\Omega d\Omega\, u^2} \int_{\xi=0}^{L_x/2} d\xi \int_\Omega d\Omega\ u(x,y,z)\, u(x+\xi,y,z)$$



Turbulence is anisotropic

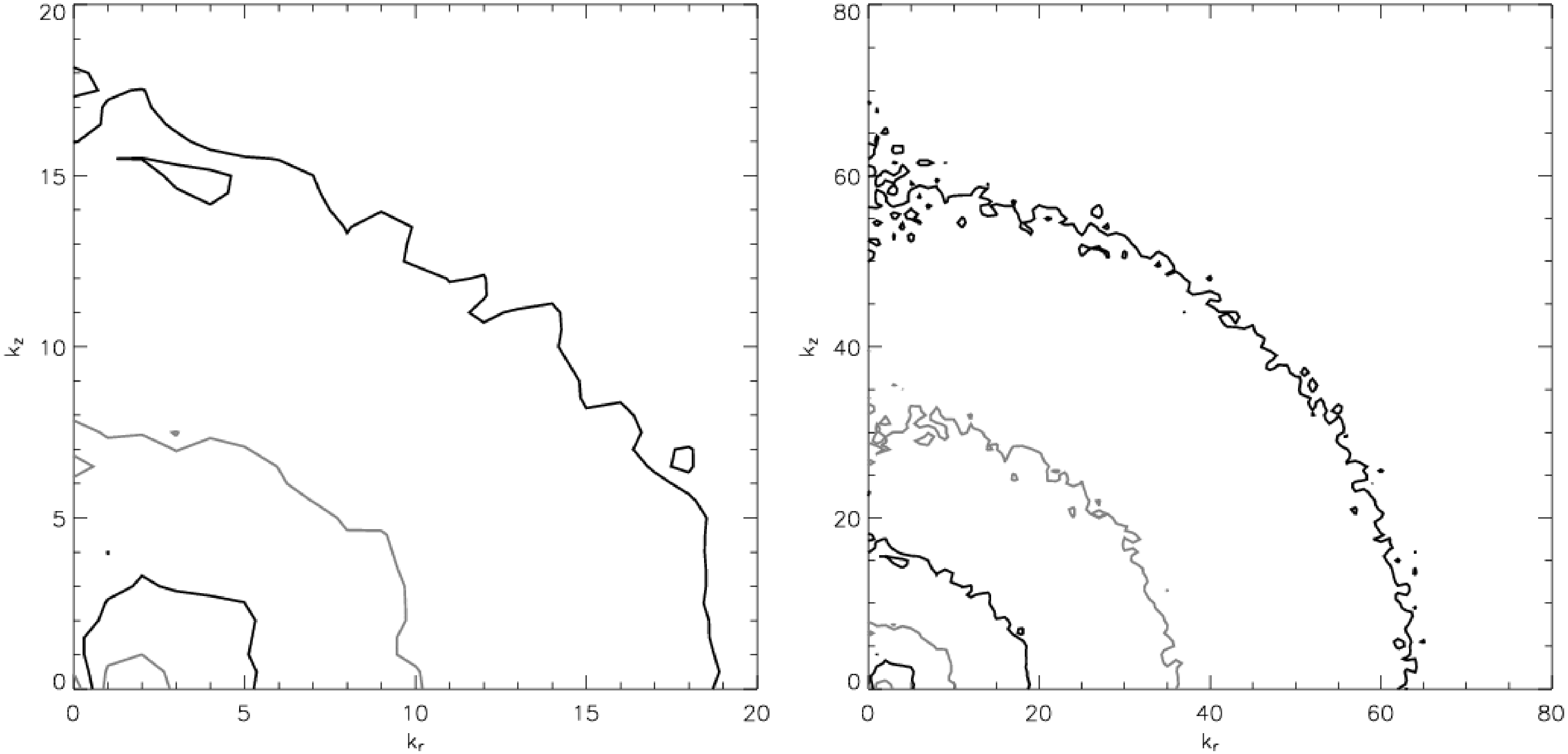
- Integral scale in z is 5x larger than in x, y
- Turbulent intensity in z is 2-3 times larger than in x,y

Gibson scale is just resolved

$$l_G = l_t \left(\frac{S_l}{u'}\right)^3$$

Turbulence on Small Scales

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655



- Look at $E(k_x,k_y,k_z)$ to see the scales it is anisotropic
  - Average over the cylindrical angle due to symmetry
  - At the largest scales (small k) we are anisotropic
  - At small scales (large k) we get circular → isotropic.

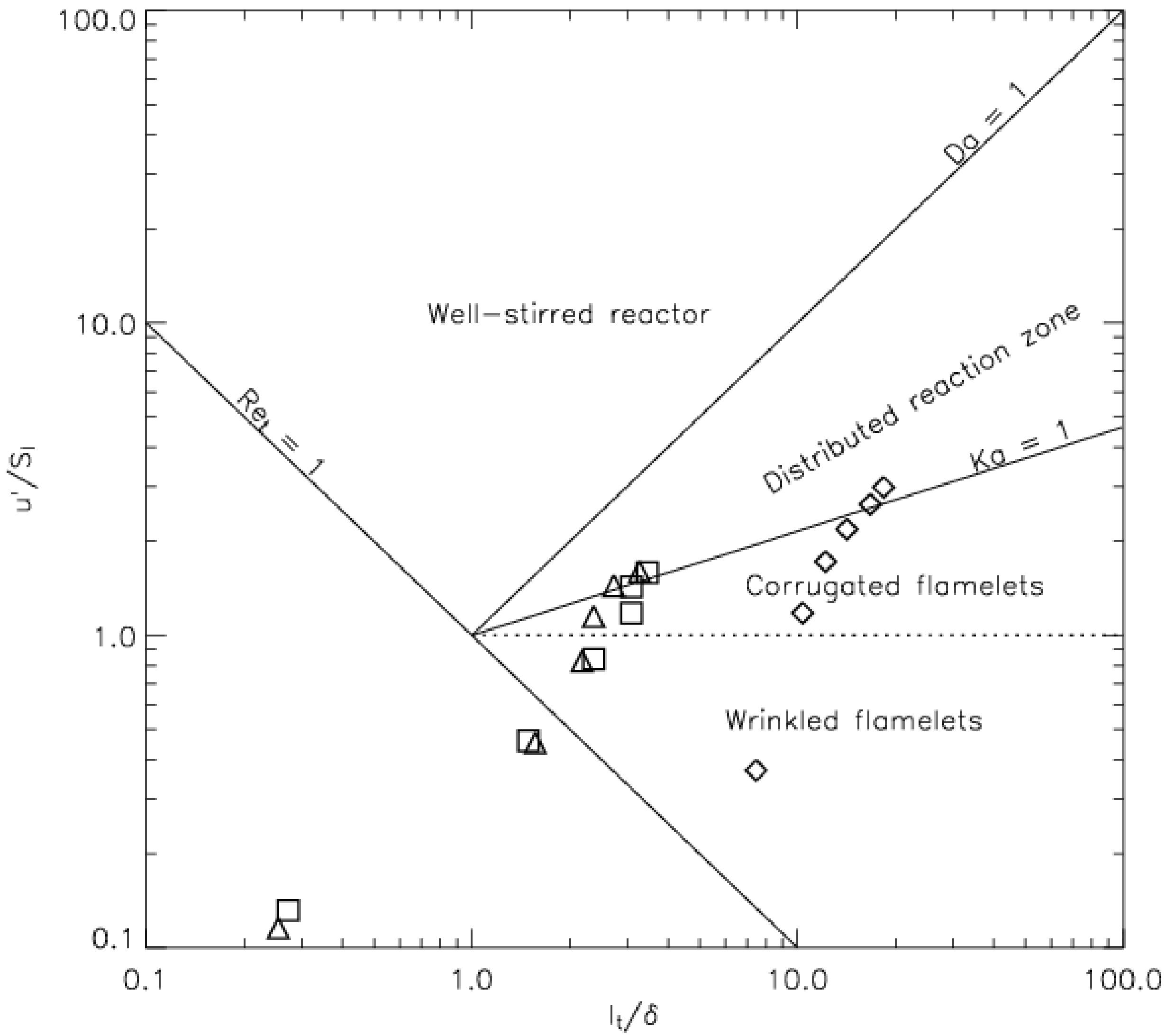
Combustion Regime

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

- Different regimes separated by lines of constant:
  - Damköhler number: integral time to reaction time (corresponds to the largest eddies)
  - Karlovitz number: reaction time to Kolmogorov time (corresponds to the smallest eddies)
  - Turbulent Reynolds number: based on integral scale
- Flamelet: Ka < 1, Da > 1
- Distributed: Ka > 1, Da > 1

Combustion Regime

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655



- As our flame progresses, we just enter the distributed reaction zone.

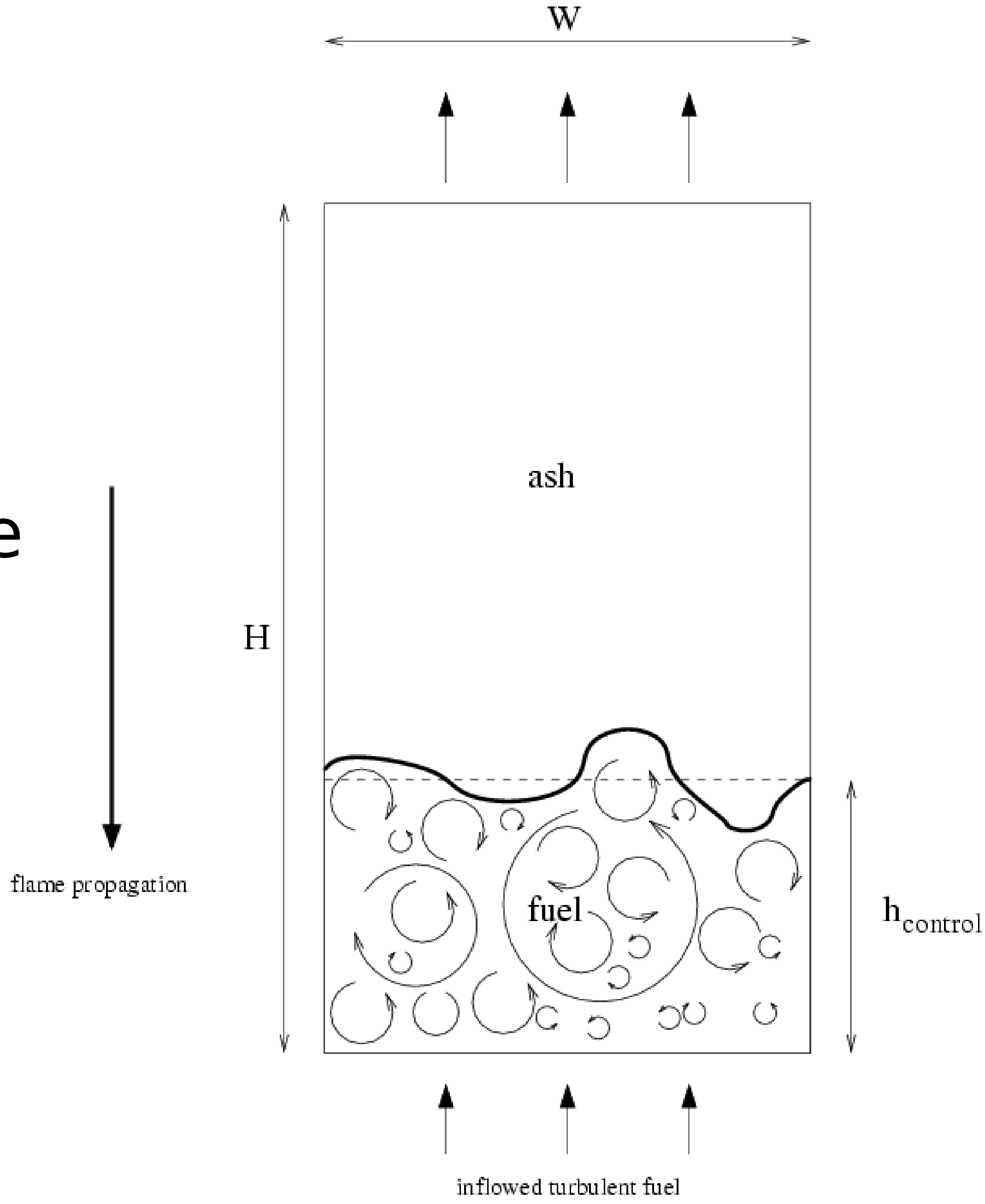
3-D Reactive RT Summary

(Zingale et al. 2005, ApJ, in press, astro-ph/0501655

- Flame width, fire-polishing length, and Gibson scale are resolved on the grid.
- Flame becomes fully turbulent.
  - Anisotropic Kolmogorov spectrum becomes isotropic after a decade of turbulent cascade.
    - Turbulent flame models assuming isotropy will need to really resolve the turbulence.
  - Transition to distributed burning regime is at a higher density in 3-D.

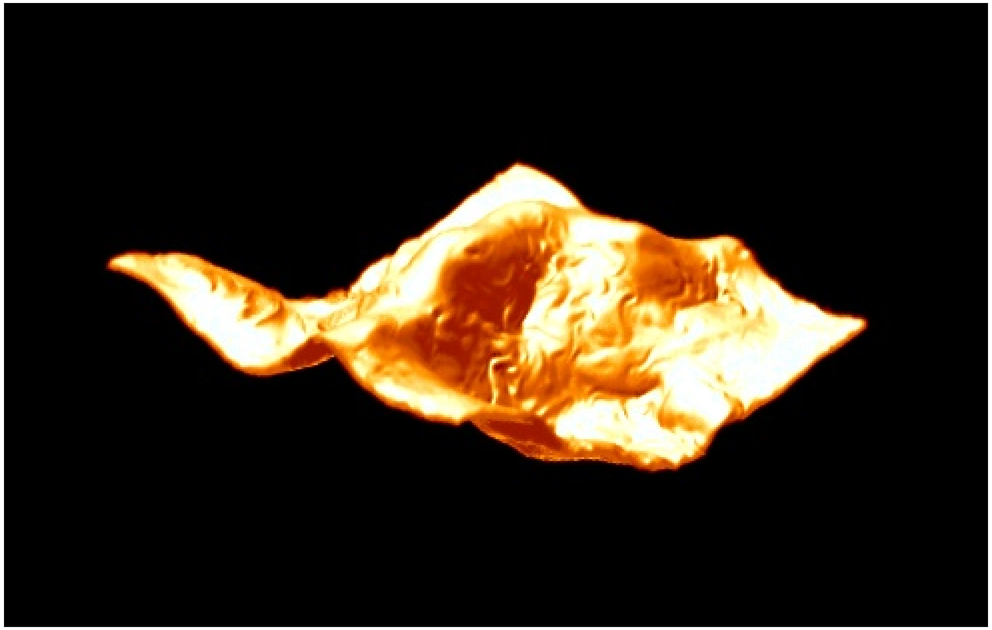
Turbulent Flames

- RT calculations consider only the turbulence on the grid
- Turbulent cascade from above can dominate
- Look at flame/turbulence interaction on scales ~ 50 flame thicknesses
  - Vary density to look at transition to distributed burning

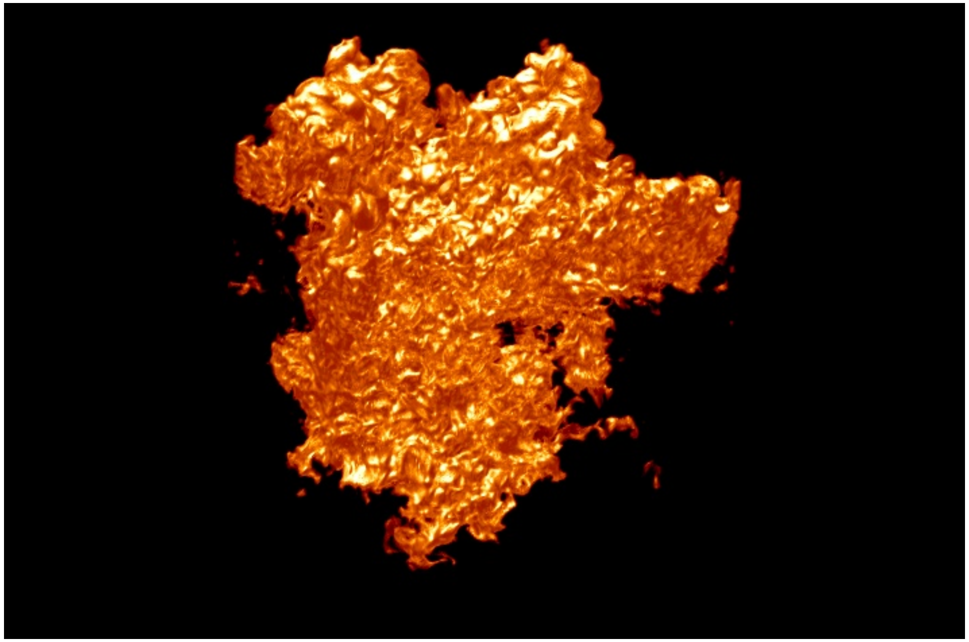


Turbulent Flames

- Parameter study underway



At high densities, the flame is smooth on the scale of the flame thickness



At low densities, the turbulence disrupts the flame structure itself.

Ignition Process

- This remains perhaps the greatest uncertainty in Type Ia supernovae models.
- Star convects for ~ 100 years.
- Highly screened carbon burning at the center
  - Ignition occurs when timescale for nuclear energy increase ~ convective turnover time (~10 s).
  - $T \sim 7 \times 10^8$ K, $\rho \sim 2 \times 10^9$ g cm^{-3}
- Does ignition occur at a single or multiple points?
  - What is the temporal distribution?
- Studies of ignition require a code suited to long time integration.

Stratified Low Mach Number Code

(Almgren et al. 2005 ApJ, in press, astro-ph/0509892)

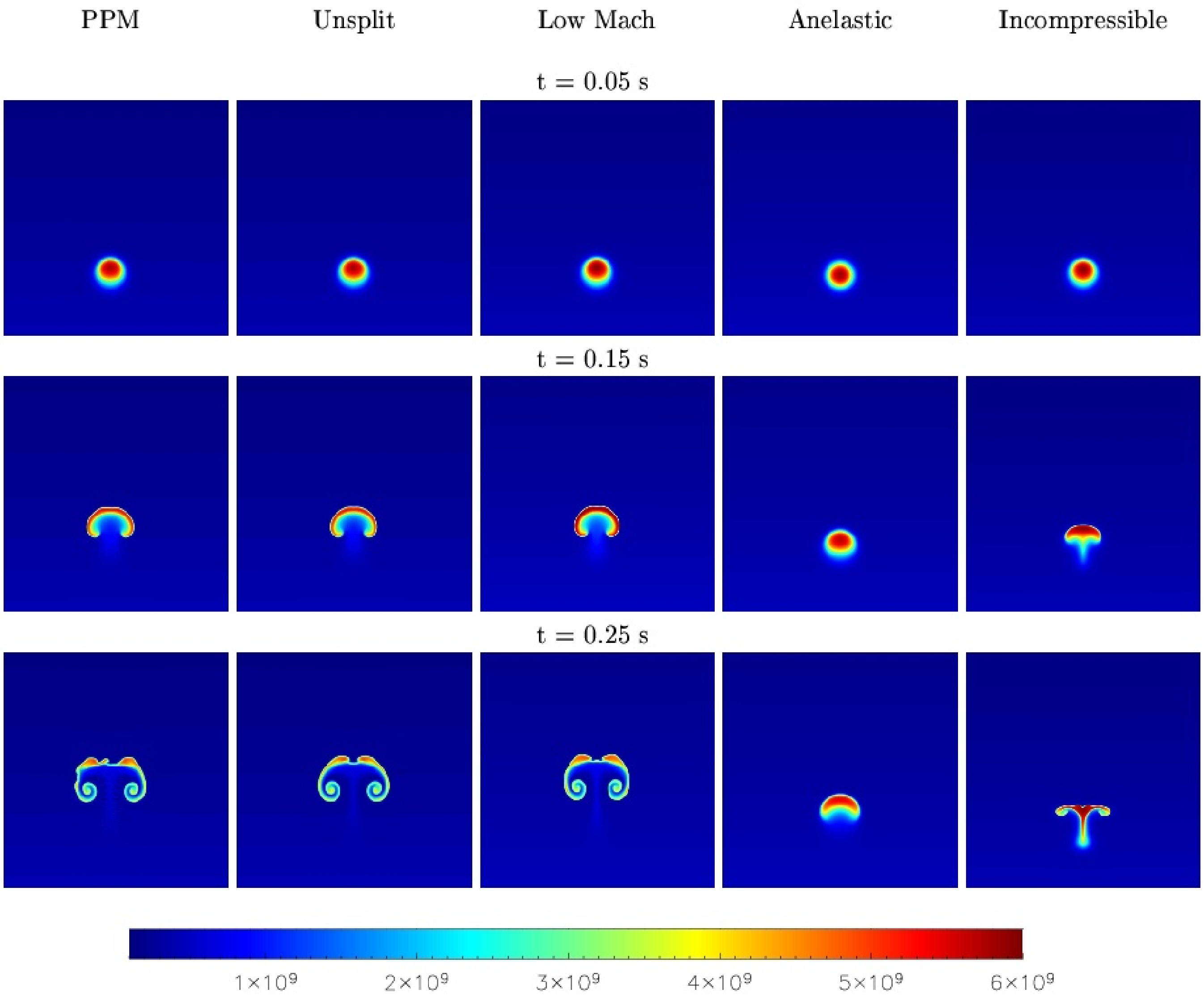
- We are extending the low Mach number methodology to the full star
  - Reformulation of the pseudo-incompressible method by Durran (1989) to general equations of state
  - Compressibility effects from both the background stratification and localized heating are incorporated
  - Pressure perturbation must be small
    - Finite amplitude density/temperature perturbations allowed

$$\nabla \cdot U + \alpha U \cdot \nabla p_0 = \frac{1}{\rho p_\rho}\left(\frac{p_T}{\rho c_p}\left(\nabla \cdot (\kappa \nabla T) - \sum_k \rho (q_k + \xi_k)\dot{\omega}_k\right) + \sum_k p_{X_k}\dot{\omega}_k\right) \equiv \tilde{S}$$

$$\alpha = \frac{1}{\Gamma_1 p_0}$$

Stratified Low Mach Number Code

(Almgren et al. 2005 ApJ, in press, astro-ph/0509892)



- Compares well to compressible codes to Mach 0.2
  - Performance gain increases as M decreases
- Work is underway to couple in reactions

Conclusions

- Transition to distributed burning at ~3×10^7 g cm^{-3}
  - Transition occurs at lower density in 2-D due to B-O scaling
- Scaling of velocity with area is not purely geometric near the transition to distributed burning
- Mixed region grows slower than Sharp-Wheeler model.
- Turbulence dominates in 3-D
  - Anisotropic Kolmogorov cascade
  - Isotropic on small scales
- Turbulent subgrid models assuming isotropy on small scales are a reasonable approximation.