Multidimensional Simulations of Convection Preceding a Type I X-ray Burst

Chris Malone

Dissertation Defense

July 27th, 2011

Outline

Introduction

Past numerical studies

The `Maestro` Code

Pure ^{4}He Accreting XRB

Mixed H/He Accreting XRB

Wrap-up

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Acknowledgements

Wetware

- Mike Zingale (SUNY Stony Brook)
- Ann Almgren, John Bell and Andy Nonaka (CCSE LBNL)
- Andrew Cumming (McGill University)
- Stan Woosley (UCSC)
- Alex Heger (UMN)

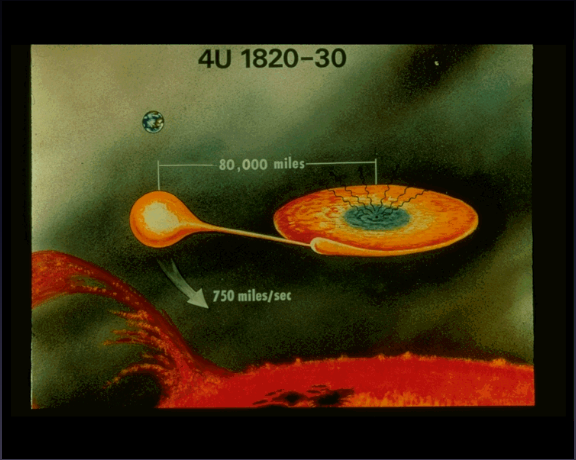
Hardware

- `franklin` and `hopper` at LBNL's NERSC
- `nyblue` at BNL
- `kraken` at NICS

What is a Type I X-ray Burst (XRB)?

Properties

- Low Mass X-ray Binaries
- Accretion of H and/or He
- Ignition at base of accreted layer



$E \sim 10^{39}$ ergs

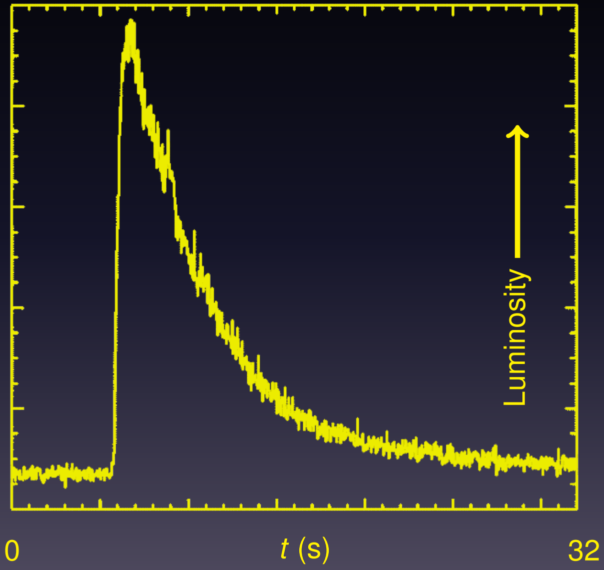
$\tau_{\text{rise}} \sim 1$ s

$\tau_{\text{dur.}} \sim 10$–$100$ s

$\tau_{\text{recur.}} \sim$ hours to days

heasarc.gsfc.nasa.gov/Images/exosat/slide_gifs/exosat18.gif

What is an XRB?: Lightcurve



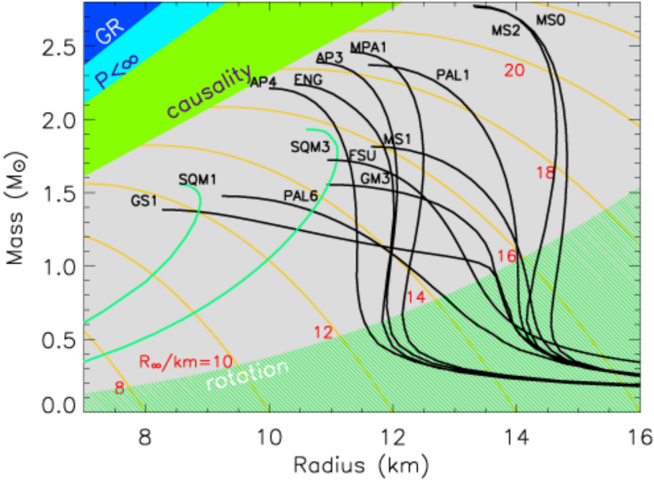
4U 1728-34

- Buring mode sets τ_{dur}.
- Inferred ignition column implies deflagration → subsonic flow

after
Strohmayer, T., et al., *ApJL*, **469**, L9, (1996)

Why do we care?

- They are explosions!
- Unique location for rp-process burning
- Flame propagation under extreme conditions
- Constrain EOS for dense matter



Lattimer, J.M., *ApSS*, **308**, 371 (2007)

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Modeling XRBs

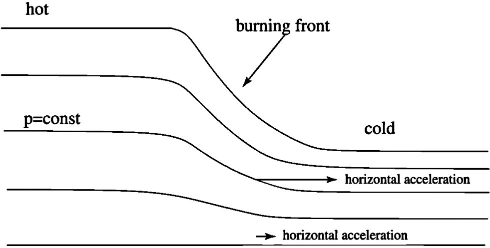
1-d

- Reproduce E, τ_{rise}, $\tau_{\mathrm{dur.}}$, $\tau_{\mathrm{recur.}}$
- Large reaction networks
- Computationally inexpensive
- Assume spherical symmetry
- Parameterized convection (MLT)
- No lateral flame propagation

Multi-d

- Simulation of multiple bursts infeasible
- Small reaction networks
- Computationally expensive
- Do not assume spherical symmetry
- No assumptions about convection
- Can model flame spreading

2-d Models of XRBs



Spitkovsky et al., *ApJ*, **566**, 1018 (2002)

- Very few!!
- Mostly treated as detonations—inconsistent with most observations
- Spitkovsky et al. used 2–layer, shallow water, incompressible, ideal gas to show importance of rotation

Modeling Type I X-ray Bursts

The problem domain is divided into zones, and discretized versions of the various conservation laws tell us how to evolve quantities in each zone over a small timestep.

Difficulty:

- Necessary condition for convergence of explicit hydrodynamics algorithm
  (CFL condition):
  Information can not propagate more than one grid zone in a single timestep.

Modeling Type I X-ray Bursts

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Compressible hydrodynamics (e.g. Euler equations)

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- Very supersonic (detonations; $M \equiv \frac{U}{c_\mathrm{s}} \gg 1$): $\Delta t \lesssim \frac{\Delta x}{U}$
- Very subsonic (deflagrations; $M \ll 1$):

$$\Delta t \lesssim \frac{\Delta x}{c_\mathrm{s}}$$

2-d, Low Mach Number Models of XRBs

Low Mach Number Approximation Methods

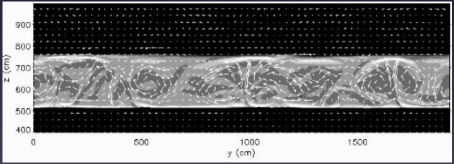
- Filter acoustics—timestep size determined by *dynamics*:

$$\Delta t \lesssim \frac{\Delta x}{U}$$

- Factor of $1/M$ increase in timestep size
- Common example: incompressible fluid — $\nabla \cdot U = 0$
- Can assume background/base state in HSE and consider motions about this state

Lin et al. low Mach number method

- Important first step
- 1^{st} order accurate in space and time
- Didn't model top of atmosphere
- Time-independent base state



Lin et al., *ApJ*, **653**, 545 (2006)

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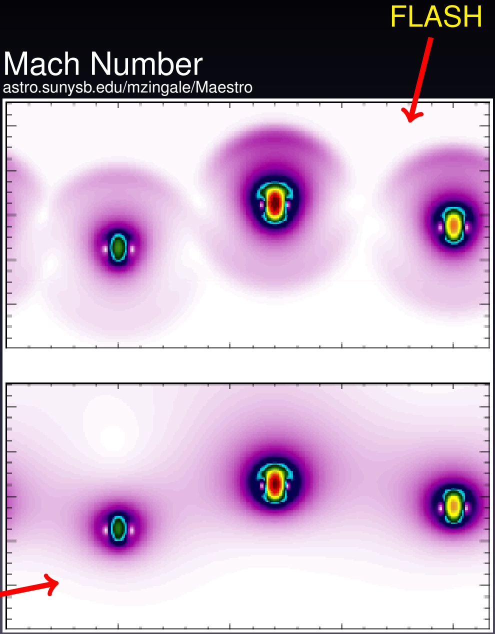
Maestro

The `Maestro` Algorithm

- Described in Nonaka et al., *ApJSS*, **188**, 358 (2010)
- Second order accurate in space and time
- Uses Adaptive Mesh Refinement
- Filters acoustics; retains compressible effects due to stratification, thermal diffusion and composition change

Mach Number

astro.sunysb.edu/mzingale/Maestro



Filering Acoustics

`Maestro` low Mach number equation set

- Decompose pressure field: $p(\boldsymbol{x}, r, t) = p_0(r, t) + \pi(\boldsymbol{x}, r, t)$

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$$\begin{aligned}
\frac{\partial(\rho X_k)}{\partial t} &= -\nabla\cdot(\rho X_k \mathbf{U}) + \rho\dot{\omega} \\
\frac{\partial \mathbf{U}}{\partial t} &= -\mathbf{U}\cdot\nabla\mathbf{U} - \frac{1}{\rho}\nabla\pi - \frac{\rho - \rho_0}{\rho} g\boldsymbol{e}_r \\
\frac{\partial(\rho h)}{\partial t} &= -\nabla\cdot(\rho h \mathbf{U}) + \frac{Dp_0}{Dt} + \rho H_{\mathrm{nuc}} + \nabla\cdot(\kappa\nabla T) \\
\nabla\cdot(\beta_0 \mathbf{U}) &= \beta_0\left(S - \frac{1}{\overline{\Gamma}_1 p_0}\frac{\partial p_0}{\partial t}\right)
\end{aligned}$$

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Initial 1-d Model Generation

Discussed in Malone et al., *ApJ*, **728**, 118 (2011)

- Construct simple atmosphere in HSE and thermal equilibrium; (similar to Cumming & Bildsten, *ApJ*, **544**, 453 (2000))

$$\begin{aligned} \frac{dT}{dy} &= \frac{3\kappa}{4acT^3}F \\ \frac{dF}{dy} &= 0 \end{aligned}$$

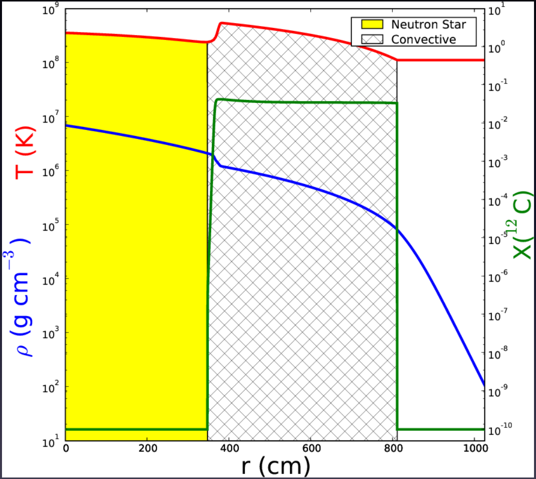
where $dy = -\rho dr$ with y as column depth

- Include constant heat flux from deep crustal heating: $F = 200$ keV / nucleon
- Base should be thermally unstable: $\frac{d\epsilon_{\rm nuc}}{dT} > \frac{d\epsilon_{\rm cool}}{dT}$
- Typically $\epsilon_{\rm cool}$ is approximated as $\epsilon_{\rm cool} \approx \frac{acT^4}{3\kappa y^2}$

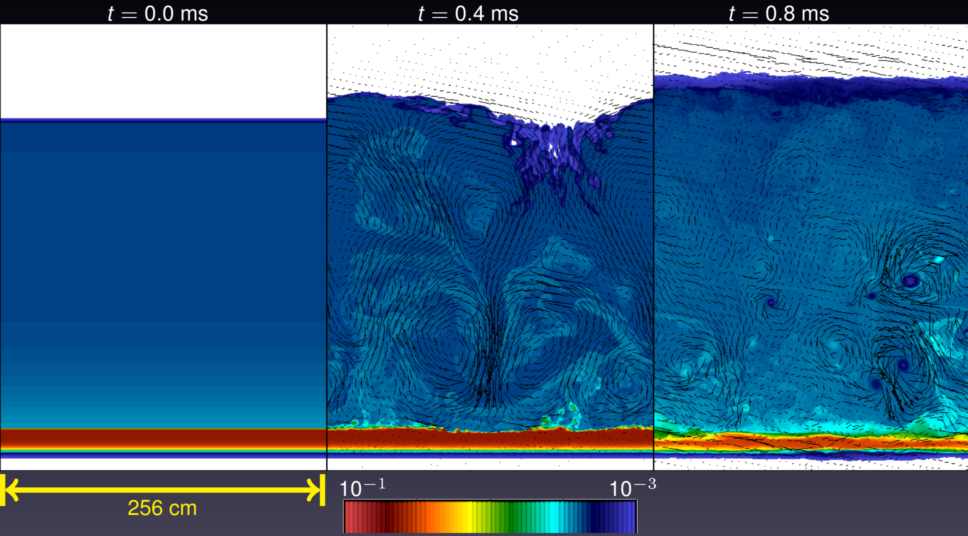
Initial 1-d Model Generation

Mapping into multi-d:

- Cooling from convective overturn
- Cooling needs to be augmented for the system to be unstable
  $\epsilon_{\rm cool} \approx \frac{acT^4}{3\kappa y^2} + \epsilon_{\rm conv.}$
- Convective cooling provided by MLT in the Kepler code (thanks to Stan Woosley)

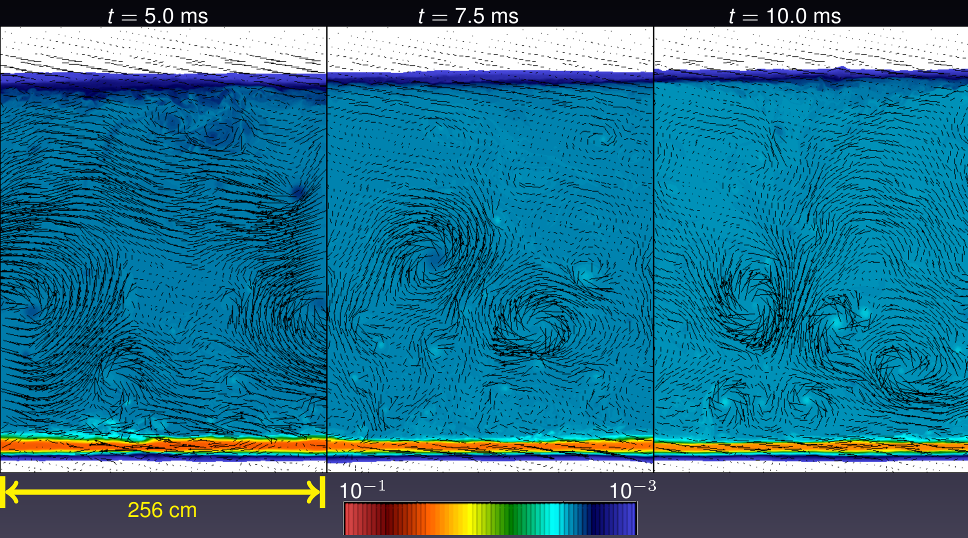


Conv. Dynamics: X(^{12}C) and $\vec{\mathbf{U}}$



$\Delta x = 0.5$ cm

Conv. Dynamics: X(^{12}C) and $\vec{\mathbf{U}}$



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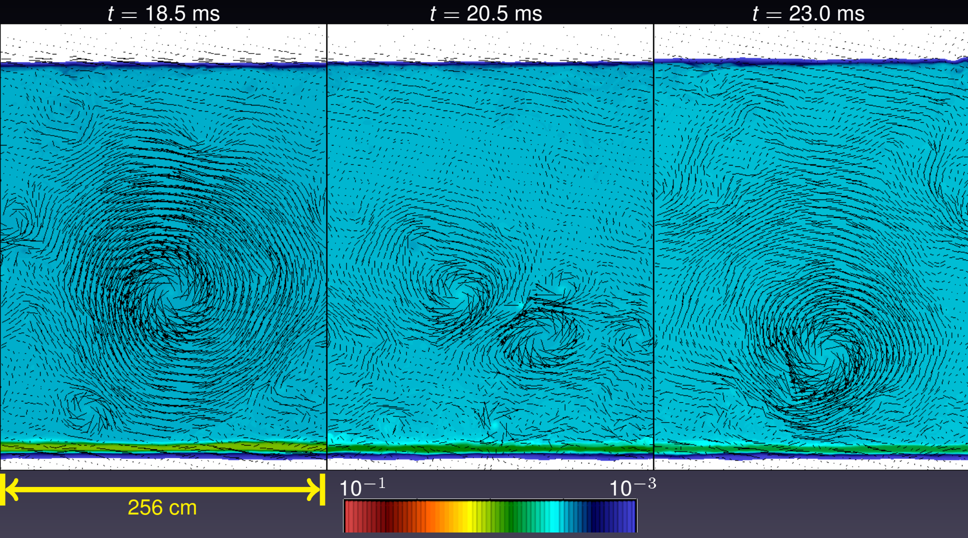

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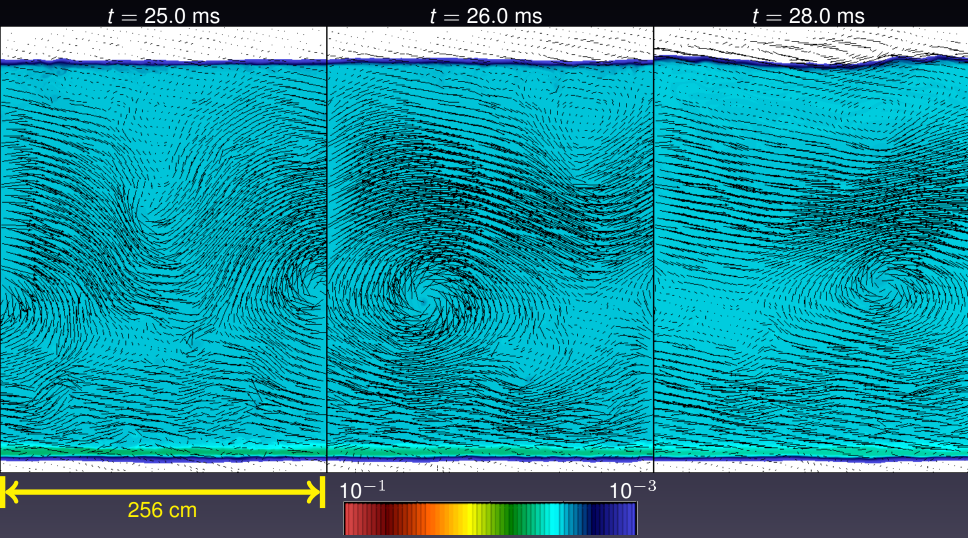
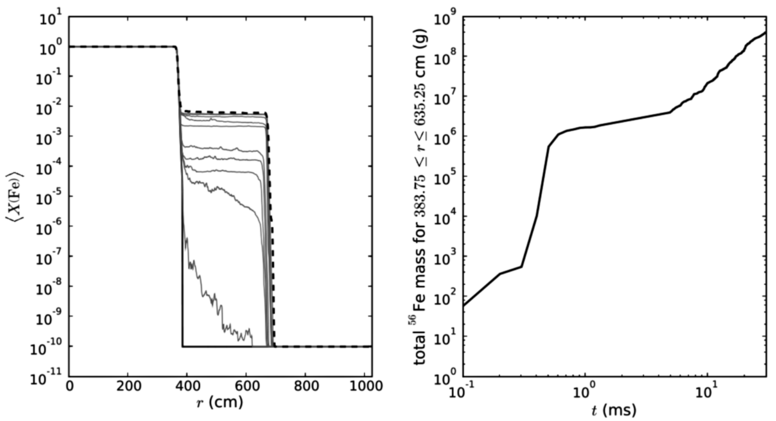


$\Delta x = 0.5$ cm

Convective Dynamics: Iron Dredge-up



- Eddies interact w/lower convective boundary; shearing
- Dredge-up of underlying neutron star material; affects conductivity!

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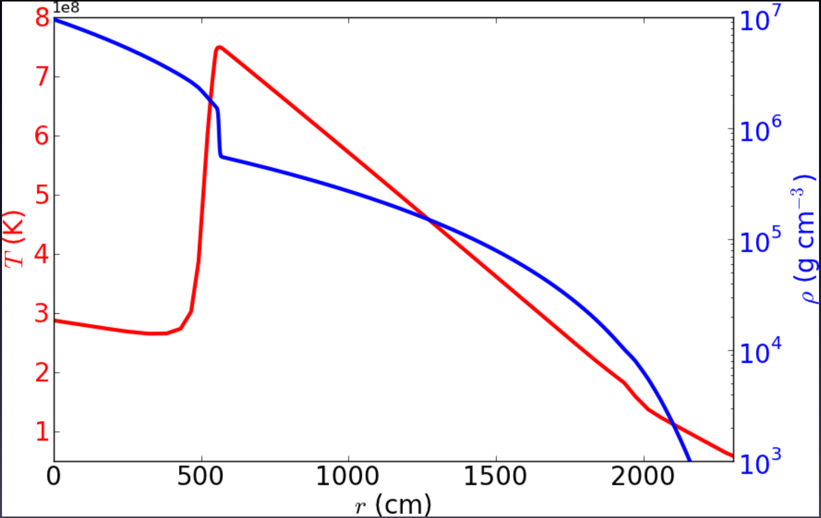
Nuclear Burning in Mixed H/He XRBs

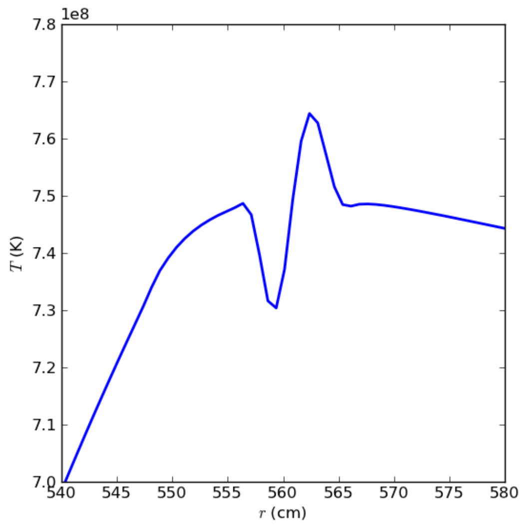
- Majority of H-burning is from hot CNO cycle
- Majority of He-burning is from 3–α reaction
- Several α-chain reactions are important: $(\alpha, \gamma), (\alpha, p)(p, \gamma)$
- rp-process burning appears: series of *p*-captures and β^+-decays to elements well beyond iron group
- Example network `hotcno` from Frank Timmes: 21 species, 52 rates. Pushing limits of what can be done in multi-d!
- We use a smaller network that approximates H-,He-,C-burning: Timmes' `aprox8` with 8 reacting species, 20 rates

Initial Model (thanks Alex Heger)



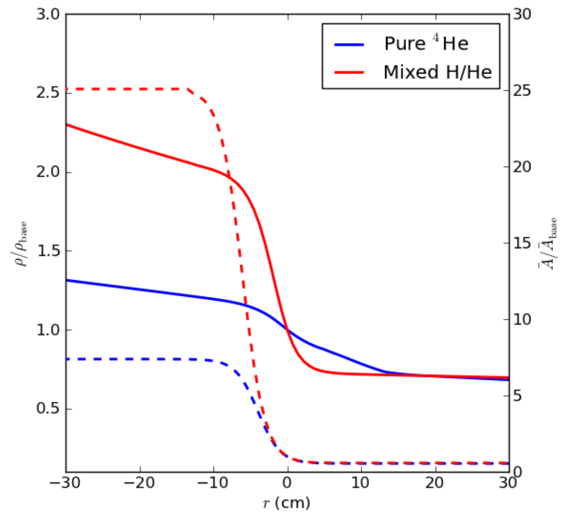
Problems with Models

- Mapped this model into `Maestro` in the same way as pure ^{4}He models
- This is what we saw after 0.2 ms of evolution



Problems with Models

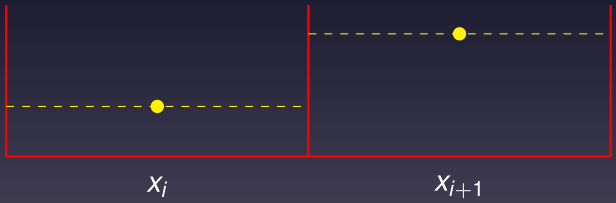
- What is different about this model compared to pure ^{4}He models?



Finite Volume Methods

Data and Evolution

- Data at cell-centers are approximate *average* of distribution over volume of grid cell
- To update a quantity in a given cell, we need to know (in part) how much is flowing in and out of the cell — i.e. the fluxes through the boundaries



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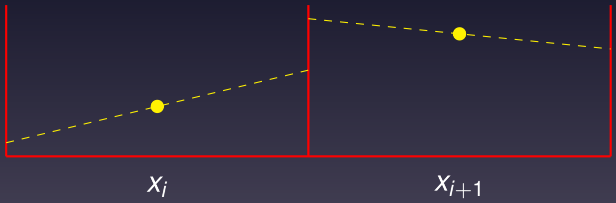


Edge-prediction

In `Maestro`

- We carry perturbational quantities, e.g. $\rho' = \rho - \rho_0$
- For fluxes in conservative update, we have a number of analytically equivalent options to reconstruct the values at the edges

Example (Edge-prediction)

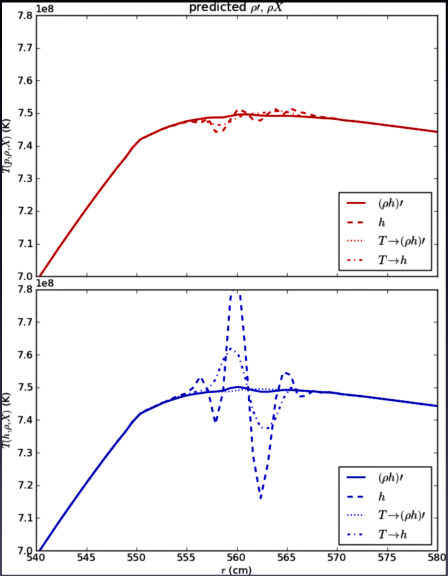
To get (ρX) at edges, we can predict:

- (ρX) explicitly to edges
- ρ and X separately to edges and combine
- ρ' and X separately to edges and combine with an average ρ_0 at edges
- …

Edge-prediction Results

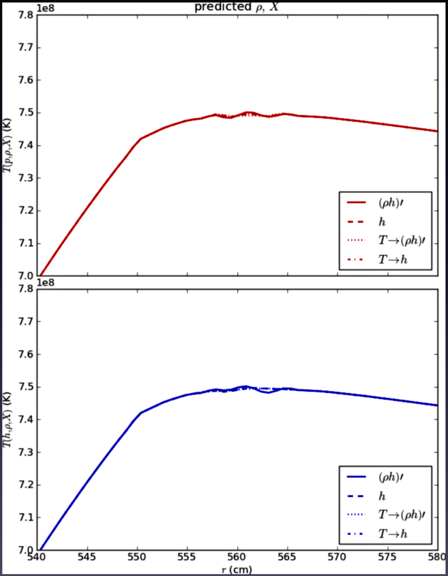
		enthalpy prediction methods			
		$(\rho h)'$	h	$T \rightarrow (\rho h)'$	$T \rightarrow h$
species prediction methods	ρ', X	bad terrible	bad so-so	bad good	bad good
	$\rho', \rho X$	good good	so-so terrible	good good	good bad
	ρ, X	good good	good good	good good	good good

Edge-prediction Results (example: row 2)



- Top (red) is $T(p, \rho, X)$
- Bottom (blue) is $T(h, \rho, X)$
- Large variation in behaviour

Current Best Approach (row 3)



- Current best choice: predict full ρ and X to the edges, plus any type of enthalpy prediction
- In theory, the results of all of these approaches should converge with increasing resolution; currently investigating…

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Conclusions

- For a system that is close to runaway in mult-d, ϵ_{cool} needs to include convective terms.

- The strong convection interacts with and churns up the underlying neutron star material—this affects conductivity and interpretation of mass and radius.
- Mixed H/He burning models are underway
- Getting a new problem to work in `Maestro` is not always trivial

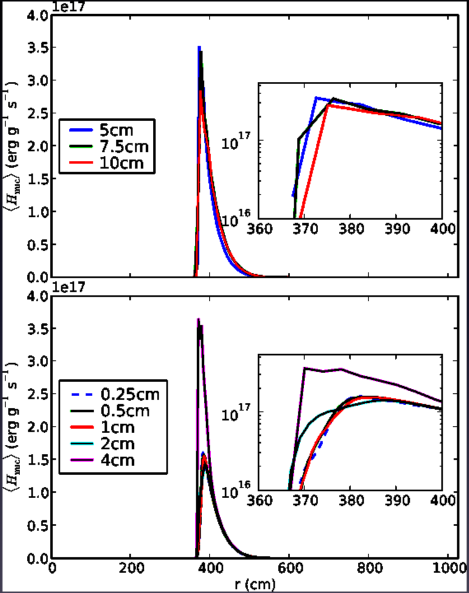
Current and Future Work

- Investigate different initial models—maybe further relax resolution requirement
- Compare 3-d runs with 2-d runs—does dredge-up still occur?
- Use tracer particles for post-processing detailed nucleosynthesis
- Add magnetic fields to `Maestro`

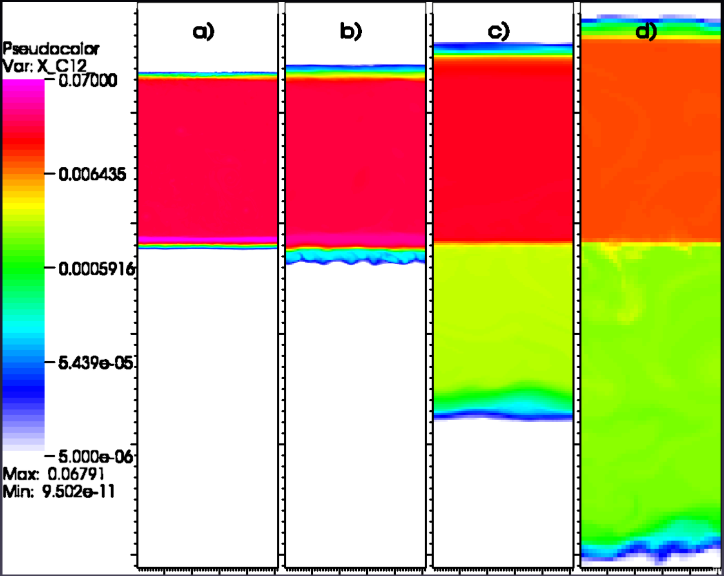
Resolution Studies

Resolution issues

- Sharp jump in T creates very peaked $H_{\rm nuc}(\sim T^{40})$
- Requires high resolution ($\sim$ 0.5 cm / zone) to resolve thin burning layer
- Comparison: Lin et al. used 5 cm / zone
  note their model used a pure ^{12}C substrate and had a much smoother thermal profile



Resolution Studies: Under-resolving=BAD!



a) 0.5 cm zone^{-1}; b) 2 cm zone^{-1}; c) 4 cm zone^{-1}; d) 7.5 cm zone^{-1}

Expansion of Atmosphere

With MAESTRO we can model the surface of the star without numerical complications.

Furthermore, we use a time-dependent base state, which allows us to capture the expansion of the atmosphere from heating.