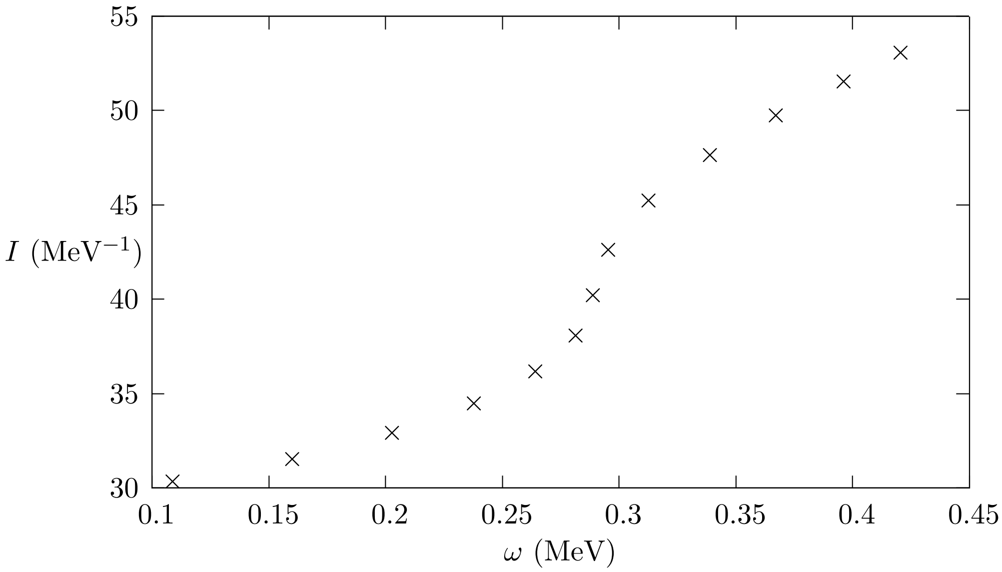
Chris Malone
Nuclear: HW # 5
March 26, 2007

QUESTION 1
For $^{158}_{66}$Dy look up energies of the rotational band based on the ground state ($J = 0 \ldots 28^+$). Plot momentum of inertia $I(J)$ (defined as $E_J = \frac{J(J+1)}{2I}$) versus rotational frequency ($\omega = \frac{\partial E_J}{\partial J} \approx \frac{E_{J+2}-E_J}{2}$). Explain the observed behavior. Compare I to that of a sphere with $R = 1.2A^{1/3}$, $A = 158$.

Using data from the NNDC at BNL, we have:

J^π	E_J (MeV)
0^+	0.0
2^+	0.0989
4^+	0.3171
6^+	0.6377
8^+	1.0439
10^+	1.5201
12^+	2.0489
14^+	2.6123
16^+	3.1904
18^+	3.7814
20^+	4.4072
22^+	5.0853
24^+	5.8200
26^+	6.6126
28^+	7.4540

$\omega \approx \frac{E_{J+2}-E_J}{2}$ (MeV)	$I(J) = \frac{J(J+1)}{2E_J}$ (MeV^{-1})
0.11	30.33
0.16	31.54
0.20	32.93
0.24	34.49
0.26	36.18
0.28	38.07
0.29	40.19
0.30	42.63
0.31	45.22
0.34	47.65
0.37	49.75
0.40	51.55
0.42	53.08



It almost seems as though there is a phase transition in the nuclei. Around $\omega \approx 0.28$ MeV, the moment of inertia changes very rapidly with little change in the rotation frequency. It could be that perhaps the nuclide starts off only slightly deformed, then as the rotation frequency increases it becomes more oblate and thus slowly increases in moment of inertia. Then at one point it has stretched so much that it becomes a thin disk, which might be marked by this sudden increase in moment of inertia. Then the radius of the disk continues to increase with increasing rotational frequency, while the thickness changes very little, thus giving rise to an increase in moment of inertia again.

To compare with a sphere, where the moment of inertia is given as $I = \frac{2}{5}MR^2$, we have for this case

$$I = \frac{2}{5}(158 \cdot 938)\left(\frac{1.2 \cdot 158^{1/3}}{197}\right)^2 \approx 64.29 \text{ MeV}^{-1}.$$

QUESTION 2

For ^{209}Pb look up all excited states with $E < 3$ MeV. Using spherical shell model levels, explain the quantum number of the ground and (as many as possible) excited states.

Again from BNL we have the following data for ^{209}Pb:

J^π	E(MeV)	J^π	E(MeV)
$9/2^+$	0.0	$7/2^+$	2.491
$11/2^+$	0.7788	$3/2^+$	2.538
$15/2^-$	1.423	$5/2^-, 7/2^-$	2.563
$5/2^+$	1.5671	$(11/2^-)$	2.589
$1/2^+$	2.0322	$5/2^-$	2.738
$1/2^-$	2.1494	$5/2^-$	2.869
$(3/2^-)$	2.319	$3/2^-$	2.904
$5/2^-, 7/2^-$	2.463	$3/2^-, 5/2^-$	2.994

The ground state is simply the double magic ^{208}Pb core plus a neutron in the next lowest lying level which is $2g_{9/2}$ which gives rise to $J = 9/2$ and the g state has positive parity, therefore the ground state is $9/2^+$. I am unsure as to why the next lowest lying level is $11/2^+$ but this seems to correspond to the $1i_{11/2}$ but should have parity $-$. The only other state which I see explained by the handout you gave in class is the $15/2^+$ which I believe corresponds to the $1j_{15/2}$ state and has the correct parity. I don't understand the notation of the states in parentheses, or the states which have two values of J^π. I assume most of these states are in the $N = 7$ states which are not shown on the diagram we have, nor could I find online. I guess my confusion comes from the fact that I would expect the state right above the ground state to be, from the handout, the $3d_{5/2}$ state which would have $J^\pi = 5/2^-$. Now, there are a lot of those that show up in the line data, but since they have higher energies, I expect them to be in states with $N > 6$.

QUESTION 3

The ds-shell single-particle energies with respect to ^{16}O core are: $\epsilon_{1d5/2} = -4.15$ MeV, $\epsilon_{2s1/2} = -3.28$ MeV, and $\epsilon_{1d3/2} = +0.93$ MeV. A particular effective interaction has the following set of two-body matrix elements:

$$\begin{aligned}
\langle 1d_{5/2}, 1d_{5/2}; J=0, T=1 | V | 1d_{5/2}, 1d_{5/2}; J=0, T=1 \rangle &= -2.0094 \text{ MeV} \\
\langle 1d_{5/2}, 1d_{5/2}; J=0, T=1 | V | 1d_{3/2}, 1d_{3/2}; J=0, T=1 \rangle &= -3.8935 \text{ MeV} \\
\langle 1d_{5/2}, 1d_{5/2}; J=0, T=1 | V | 2s_{1/2}, 2s_{1/2}; J=0, T=1 \rangle &= -1.3225 \text{ MeV} \\
\langle 1d_{3/2}, 1d_{3/2}; J=0, T=1 | V | 1d_{3/2}, 1d_{3/2}; J=0, T=1 \rangle &= -0.8119 \text{ MeV} \\
\langle 1d_{3/2}, 1d_{3/2}; J=0, T=1 | V | 2s_{1/2}, 2s_{1/2}; J=0, T=1 \rangle &= -0.8385 \text{ MeV} \\
\langle 2s_{1/2}, 2s_{1/2}; J=0, T=1 | V | 2s_{1/2}, 2s_{1/2}; J=0, T=1 \rangle &= -2.3068 \text{ MeV}
\end{aligned}$$

a Calculate the ground state binding energy of ^{18}O with respect to ^{16}O and compare the result obtained from a table of mass excess. What are the excitation energies of the two other 0^+ states in this space?

b Obtain the ground state wave function of ^{18}O and use it to calculate the relative probability for finding a neutron in the $1d_{5/2}, 2s_{1/2}$, and $1d_{3/2}$ single-particle states in ^{18}O. The results are essentially the spectroscopic factors for one-neutron pickup reactions.

a For this problem we start by writing the Hamiltonian of the system as

$$H = H_1 + H_2 + V_{12}$$

where $H_{1,2}$ are the interactions of particle (1,2) with the ^{16}O core and V_{12} is the two body interaction given by the matrix elements above. We then construct the Hamiltonian:

$$\begin{aligned}
H &= \begin{pmatrix} 2\epsilon_{d5/2} + V_{d5/2,d5/2} & V_{d5/2,s1/2} & V_{d5/2,d3/2} \\ V_{s1/2,d5/2} & 2\epsilon_{s1/2} + V_{s1/2,s1/2} & V_{s1/2,d3/2} \\ V_{d3/2,d5/2} & V_{d3/2,s1/2} & 2\epsilon_{d3/2} + V_{d3/2,d3/2} \end{pmatrix} \\
&= \begin{pmatrix} -10.31 & -1.32 & -3.89 \\ -1.32 & -8.87 & -0.84 \\ -3.89 & -0.84 & 1.05 \end{pmatrix} = \begin{pmatrix} -12.20 & 0 & 0 \\ 0 & -8.20 & 0 \\ 0 & 0 & 2.27 \end{pmatrix}
\end{aligned}$$

Therefore, the binding energy of the ground state (1d5/2) of ^{18}O with respect to the ^{16}O core is -12.20. Using the equation of mass excess from the BNL website, we have

$$\begin{aligned}
B(Z,N) &= Z(m_p + m_e) + Nm_n - M(Z,N) = Z(m_p + m_e) + Nm_n - \Delta - A * amu. \\
B(8,8) &= 8(m_p + m_e) + 8m_n + 4.737 - 16 * amu \\
B(8,10) &= 8(m_p + m_e) + 10m_n + 0.7815 - 18 * amu \\
B(8,8) - B(8,10) &= 4.737 - 0.7815 - 2 * (m_n - amu) \approx -12.18 \text{ MeV}
\end{aligned}$$

which is very close to the value that we get from the previous method.

The excitation energies from the ground state of ^{18}O are then

$$\begin{aligned}
\Delta E_{2s1/2} &= -8.20 + 12.20 = 4.00 \text{ MeV} \\
\Delta E_{1d3/2} &= 2.27 + 12.20 = 14.47 \text{ MeV}
\end{aligned}$$

b If I take the above mentioned matrix and find the eigenvectors using Maple, I get:

$$\lambda = -12.20 \quad : \quad \psi = \begin{pmatrix} 0.87 \\ 0.41 \\ 0.28 \end{pmatrix}$$

$$\lambda = -8.20 \quad : \quad \psi = \begin{pmatrix} -0.41 \\ 0.91 \\ -0.09 \end{pmatrix}$$

$$\lambda = 2.27 \quad : \quad \psi = \begin{pmatrix} -0.29 \\ -0.04 \\ 0.96 \end{pmatrix}$$

which seem to be normalized to unity. So the ground state wavefunction is

$$|\psi_g\rangle = 0.87\,|1d5/2\rangle + 0.41\,|2s1/2\rangle + 0.28\,|1d3/2\rangle$$

and therefore

$$\begin{aligned}
\langle 1d5/2|\psi_g\rangle &\approx 0.76 \\
\langle 2s1/2|\psi_g\rangle &\approx 0.17 \\
\langle 1d3/2|\psi_g\rangle &\approx 0.08
\end{aligned}$$